

Internet Appendix for
“Option-Implied Risk Premia
with Intertemporal Hedging”

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ABSTRACT

This Internet Appendix provides proofs and derivations of results in the main text, extensions to the models presented in Section [I](#), as well as additional empirical tests.

I . Proofs and derivations of Section I in the main text

This section contains the proofs and derivations of the results presented in Section I.

A. Proof of Equation (1)

Let R_{k,t,T_1} be the return of risky asset k from time t to time T_1 and m_{t,T_1} be the one-period SDF. We show that the conditional expected return of risky assets can be expressed as the risk-neutral covariance between the asset return and the inverse of the SDF m_{t,T_1} . This result is not new, and was derived in Equation (2) of Chabi-Yo and Loudis (2020).

The conditional expected return of asset k can be expressed using the identity

$$\mathbb{E}_t(R_{k,t,T_1}) = \mathbb{E}_t\left(R_{k,t,T_1} \frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \frac{m_{t,T_1}}{\mathbb{E}_t m_{t,T_1}}\right). \quad (\text{I1})$$

The ratio $\frac{m_{t,T_1}}{\mathbb{E}_t m_{t,T_1}}$ defines the risk-neutral distribution. Hence, the Radon-Nykodym theorem allows us to express the conditional expected return of asset k as a function of moments under the risk-neutral measure:

$$\begin{aligned} \mathbb{E}_t(R_{k,t,T_1}) &= \mathbb{E}_t^* \left(R_{k,t,T_1} \frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \right) \\ &= \text{COV}_t^* \left(\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}}, R_{k,t,T_1} \right) + \underbrace{\mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \right)}_{=1} \mathbb{E}_t^*(R_{k,t,T_1}) \\ &= \text{COV}_t^* \left(\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}}, R_{k,t,T_1} \right) + R_{f,t,T_1}. \end{aligned} \quad (\text{I2})$$

This identity is reminiscent of the well-known asset pricing equation in which the expected excess return is negatively related to the covariance between the return and the SDF under the physical measure.

B. Proof of Equation (3)

We show that the inverse of the one-period SDF m_{t,T_1} can be expressed as a function of the marginal utility of wealth and expectations under the risk-neutral measure.

The representative agent's optimization problem can be re-written as

$$\max_{\omega_t} \mathbb{E}_t (u [W_{T_N}]) = \max_{\omega_t} \mathbb{E}_t \left(\max_{\omega_{T_1}} \mathbb{E}_{T_1} (u [W_{T_N}]) \right). \quad (\text{I3})$$

Solving Problem (I3) backward, the first step is to solve

$$\max_{\omega_{T_1}} \mathbb{E}_{T_1} (u [W_{T_N}]). \quad (\text{I4})$$

Equation (I4) produces an optimal weight $\omega_{T_1}^*$, and the terminal wealth achieved with this weight is $W_{T_N}^* = W_{T_1} (\omega_{T_1}^{*\top} R_{T_1, T_N})$. The corresponding one-period SDF from time T_1 to time T_N , m_{T_1, T_N} , has the form

$$m_{T_1, T_N} = \delta_{T_1} u' [W_{T_N}^*]. \quad (\text{I5})$$

Given the optimal value, $\omega_{T_1}^*$, the second step solves

$$\max_{\omega_t} \mathbb{E}_t (\mathbb{E}_{T_1} (u [W_{T_N}^*])). \quad (\text{I6})$$

This produces a one-period SDF from time t to time T_1 of the form

$$m_{t,T_1} = \delta_t \mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right). \quad (\text{I7})$$

Equation (I7) gives us the constant δ_t :

$$\delta_t = m_{t,T_1} \left(\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right) \right)^{-1}. \quad (\text{I8})$$

Because parameter δ_t is a constant, we have $\delta_t = \mathbb{E}_t \delta_t$. We exploit the no-arbitrage conditions that allow us to move from the physical measure to the risk-neutral measure to obtain,

$$\begin{aligned} \delta_t &= \mathbb{E}_t \left(m_{t,T_1} \left(\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right) \right)^{-1} \right) \\ &= \mathbb{E}_t (m_{t,T_1}) \mathbb{E}_t \left(\frac{m_{t,T_1}}{\mathbb{E}_t (m_{t,T_1})} \left(\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right) \right)^{-1} \right) \\ &= \mathbb{E}_t (m_{t,T_1}) \mathbb{E}_t^* \left(\left(\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right) \right)^{-1} \right). \end{aligned} \quad (\text{I9})$$

Next, we replace δ_t by its expression in (I8) and obtain

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{1/\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right)}{\mathbb{E}_t^* \left(1/\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right) \right)}. \quad (\text{I10})$$

Similarly, we can use the SDF (I5) and show that

$$\frac{\mathbb{E}_{T_1} m_{T_1,T_N}}{m_{T_1,T_N}} = \frac{1/u' [W_{T_N}^*]}{\mathbb{E}_{T_1}^* \left(1/u' [W_{T_N}^*] \right)}. \quad (\text{I11})$$

Next, we write $\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1,T_N}) \right)$ in (I10) as a function of risk-

neutral quantities:

$$\begin{aligned}
\mathbb{E}_{T_1} \left(u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1, T_N}) \right) &= \mathbb{E}_{T_1} \left(\frac{m_{T_1, T_N}}{\mathbb{E}_{T_1} m_{T_1, T_N}} \frac{\mathbb{E}_{T_1} m_{T_1, T_N}}{m_{T_1, T_N}} u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1, T_N}) \right) \\
&= \mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_{T_1} m_{T_1, T_N}}{m_{T_1, T_N}} u' [W_{T_N}^*] (\omega_{T_1}^{*\top} R_{T_1, T_N}) \right) \\
&= \frac{\omega_{T_1}^{*\top} \mathbb{E}_{T_1}^* R_{T_1, T_N}}{\mathbb{E}_{T_1}^* (1/u' [W_{T_N}^*])} = \frac{R_{f, T_1, T_N}}{\mathbb{E}_{T_1}^* (1/u' [W_{T_N}^*])}, \tag{I12}
\end{aligned}$$

where we have used the no-arbitrage conditions to move from the physical measure to the risk-neutral measure in the second equation, and Equation (I11) to obtain the third equation.

We replace (I12) in (I10) to obtain

$$\begin{aligned}
\frac{\mathbb{E}_t m_{t, T_1}}{m_{t, T_1}} &= \frac{\frac{\mathbb{E}_{T_1}^* \left(\frac{1}{u' [W_{T_N}^*]} \right)}{R_{f, T_1, T_N}}}{\mathbb{E}_t^* \left(\frac{\mathbb{E}_{T_1}^* \left(\frac{1}{u' [W_{T_N}^*]} \right)}{R_{f, T_1, T_N}} \right)} \\
&= \frac{((1/R_{f, T_1, T_N}) / \mathbb{E}_t (1/R_{f, T_1, T_N})) \mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f, t, T_N}]}{u' [W_{T_N}^*]} \right)}{\mathbb{E}_t^* \left(((1/R_{f, T_1, T_N}) / \mathbb{E}_t (1/R_{f, T_1, T_N})) \mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f, t, T_N}]}{u' [W_{T_N}^*]} \right) \right)}. \tag{I13}
\end{aligned}$$

Since there is no interest rate risk, $1/R_{f, T_1, T_N} = \mathbb{E}_t (1/R_{f, T_1, T_N})$, and (I13) simplifies to

$$\frac{\mathbb{E}_t m_{t, T_1}}{m_{t, T_1}} = \frac{\mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f, t, T_N}]}{u' [W_{T_N}^*]} \right)}{\mathbb{E}_t^* \left(\mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f, t, T_N}]}{u' [W_{T_N}^*]} \right) \right)}. \tag{I14}$$

Assume that the gross return on the market can be used as proxy for the return on aggregate wealth:

$$R_{M,t,T_N} = \frac{W_{T_N}^*}{W_t} \quad \text{and} \quad R_{M,T_1,T_N} = \frac{W_{T_N}^*}{W_{T_1}} \quad (\text{I15})$$

Equation (I14) can be rewritten as

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{\mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f,t,T_N}]}{u' [W_t R_{M,t,T_N}]} \right)}{\mathbb{E}_t^* \left(\mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f,t,T_N}]}{u' [W_t R_{M,t,T_N}]} \right) \right)}. \quad (\text{I16})$$

This ends the proof.

C. Proof of Equation (4)

We detail below the second-order expansion of the inverse of the marginal utility, which we use to derive Proposition 1.

Define the function

$$f [x, y] = \frac{u' [W_t x_0 y_0]}{u' [W_t x y]}, \quad (\text{I17})$$

with $x = R_{M,t,T_1}$, $y = R_{M,T_1,T_N}$, $x_0 = R_{f,t,T_1}$ and $y_0 = R_{f,T_1,T_N}$. Since we assume no interest rate risk, $R_{M,t,T_N} = xy$ and $R_{f,t,T_N} = x_0 y_0$.

We write the inverse of the SDF as

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{\mathbb{E}_{T_1}^* (f [x, y])}{\mathbb{E}_t^* (\mathbb{E}_{T_1}^* (f [x, y]))}. \quad (\text{I18})$$

We adopt the following short notations. First, we use f_x and f_y to denote

the first partial derivatives of the function f , f_{xx} and f_{yy} the second partial derivatives, and f_{xy} the cross-derivative, all evaluated at (x_0, y_0) . Second, we denote as u' , u'' , and u''' the first, second, and third derivatives of $u[\cdot]$ evaluated at (x_0, y_0) . We perform a second-order Taylor expansion series of $f[x, y]$ around $(x, y) = (x_0, y_0)$:

$$f[x, y] \approx 1 + \frac{1}{1!} (x - x_0) f_x + \frac{1}{1!} (y - y_0) f_y + \frac{1}{2!} (x - x_0)^2 f_{xx} + \frac{1}{2!} (y - y_0)^2 f_{yy} + \frac{2}{2!} (x - x_0) (y - y_0) f_{xy}, \quad (\text{I19})$$

where

$$f_x = \frac{y_0}{x_0} f_y = \frac{1}{x_0} \left(-\frac{(W_t x_0 y_0) u''}{u'} \right), \quad (\text{I20})$$

$$f_{xy} = \frac{1}{x_0 y_0} \left(-\frac{W_t x_0 y_0 u''}{u'} \right) + \frac{1}{x_0 y_0} \frac{(W_t x_0 y_0 u'')^2}{(u')^2} \left(2 - \frac{u''' u'}{(u'')^2} \right) \text{ and} \quad (\text{I21})$$

$$f_{xx} = \frac{y_0^2}{x_0^2} f_{yy} = \frac{1}{(x_0)^2} \frac{(W_t x_0 y_0 u'')^2}{(u')^2} \left(2 - \frac{u''' u'}{(u'')^2} \right). \quad (\text{I22})$$

Implicit in this approach is the assumption that the expansion provides a good local approximation in the neighborhood of $(x, y) = (x_0, y_0)$.¹

¹The approximation may lose accuracy in the tails of the distribution. One possible refinement would be to partition the support of x (and similarly y) into three regions: the left tail, the center (which includes the safe return), and the right tail. This would produce nine partitions in total, with a separate expansion in each region (as in Chabi-Yo and Loudis (2024), in a static one-period model). While theoretically feasible, this approach introduces up to nine parameters, which could be reduced to five if continuity and differentiability of the inverse marginal utility are imposed. Although possible, the empirical burden of additional parameters is substantial, so we leave this extension to

Note that $f_{xy} = f_{yx}$. Thus, we obtain,

$$\begin{aligned}
f[x, y] &\approx 1 + \frac{1}{x_0} \frac{1}{\tau_t} (x - x_0) + \frac{1}{y_0} \frac{1}{\tau_t} (y - y_0) \\
&\quad + \frac{1}{(x_0)^2} \frac{(1 - \rho_t)}{\tau_t^2} (x - x_0)^2 + \frac{1}{(y_0)^2} \frac{(1 - \rho_t)}{\tau_t^2} (y - y_0)^2 \\
&\quad + \frac{1}{x_0 y_0} \left(\frac{1}{\tau_t} + \frac{2(1 - \rho_t)}{\tau_t^2} \right) (x - x_0) (y - y_0), \tag{I23}
\end{aligned}$$

where τ_t and ρ_t are defined in Equations (7) and (8). Replacing x , x_0 , y , and y_0 by their expressions and using preference parameters $a_{1,t}$ and $a_{2,t}$ defined in (6), we obtain

$$\begin{aligned}
\mathbb{E}_{T_1}^* (f[x, y]) &\approx 1 + \frac{a_{1,t}}{R_{f,t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1}) + \frac{a_{1,t}}{R_{f,T_1,T_N}} \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}) \\
&\quad + \frac{a_{2,t}}{(R_{f,t,T_1})^2} (R_{M,t,T_1} - R_{f,t,T_1})^2 + \frac{a_{2,t}}{(R_{f,T_1,T_N})^2} \mathbb{E}_{T_1}^* ((R_{M,T_1,T_N} - R_{f,T_1,T_N})^2) \\
&\quad + \frac{a_{1,t} + 2a_{2,t}}{R_{f,t,T_N}} (R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}). \tag{I24}
\end{aligned}$$

Using the martingale condition $\mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}) = 0$, (I24) simplifies to

$$\mathbb{E}_{T_1}^* f[x, y] = \mathbb{E}_{T_1}^* \left(\frac{u' [W_t R_{f,t,T_1} R_{f,T_1,T_N}]}{u' [W_t R_{M,t,T_1} R_{M,T_1,T_N}]} \right) \approx 1 + z_{T_1}, \tag{I25}$$

where z_{T_1} is given by Equation (5). We then replace Equation (I25) in (I18) to obtain Equation (4).

future research.

D. Proof of Proposition 1

We use the expression for the SDF (4) derived in Section I.C, and plug it in the expected return expression identity (1). We obtain

$$\begin{aligned}
\mathbb{E}_t(R_{M,t,T_1} - R_{f,t,T_1}) &\approx \text{COV}_t^* \left(R_{M,t,T_1}, \frac{1 + z_{T_1}}{1 + \mathbb{E}_t^* z_{T_1}} \right) \\
&\approx \text{COV}_t^* \left(R_{M,t,T_1} - R_{f,t,T_1}, \frac{1 + z_{T_1}}{1 + \mathbb{E}_t^* z_{T_1}} \right) \\
&\approx \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1}) \frac{1 + z_{T_1}}{1 + \mathbb{E}_t^* z_{T_1}} \right) \quad (\text{I26})
\end{aligned}$$

We then substitute (5) in (I26) and obtain the estimate for the market risk premium in Equation (9).

E. Model with multiple portfolio rebalancing times

The results presented in the previous subsections rely on the assumption that the representative agent can rebalance their portfolio only at time T_1 . We now relax this restriction and allow the agent to rebalance at discrete times T_j , $1 \leq j < N$. Gross market and risk-free returns between t and T_N , with $t = T_0 < T_1$, can be written as

$$R_{M,t,T_N} = \prod_{j=1}^N R_{M,T_{j-1},T_j} \text{ and } R_{f,t,T_N} = \prod_{j=1}^N R_{f,T_{j-1},T_j}.$$

Let us denote $x_j = R_{M,T_{j-1},T_j}$ and $x_{0,j} = R_{f,T_{j-1},T_j}$. A second-order Taylor expansion-series of the inverse marginal utility around $(x_1, \dots, x_N) =$

$(x_{0,1}, \dots, x_{0,N})$ yields

$$z_{T_1} = \frac{a_{1,t}}{x_{0,1}} (x_1 - x_{0,1}) + \frac{a_{2,t}}{x_{0,1}^2} (x_1 - x_{0,1})^2 + a_{2,t} \sum_{j>1}^N \frac{1}{x_{0,j}^2} \mathbb{E}_{T_1}^* (x_j - x_{0,j})^2. \quad (\text{I27})$$

We replace expression (I27) in (3) and plug it in (1), to derive the equity risk premium in Equation (12).

F. Comparison of the models of Chabi-Yo and Loudis (2020) and Tetlock, McCoy, and Shah (2024)

In this subsection, we show that the SDF of Chabi-Yo and Loudis (2020) nests as a special case the SDF of Tetlock, McCoy, and Shah (2024). This result was also shown in parallel work by Vidal Nunes, Ruas, and Dias (2025).

In the model of Tetlock, McCoy, and Shah (2024), the representative agent can trade the market as well as $K - 1$ derivatives securities, with payoffs equal to the higher-order moments of the market returns. The excess return on the market is $\tilde{R}_{M,t,T_1} = R_{M,t,T_1} - R_{f,t,T_1}$ and the excess returns on derivatives securities are $\tilde{R}_{M,t,T_1}^k - c_k$, with $c_k = \mathbb{E}_t^* \left(\tilde{R}_{M,t,T_1}^k \right)$.

Equation (9) of Tetlock, McCoy, and Shah (2024) gives the inverse SDF as follows:

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = 1 + \sum_{k=1}^K \frac{1}{R_{f,t,T_1}} \omega_{k,t} \left(\tilde{R}_{M,t,T_1}^k - c_k \right) \quad (\text{I28})$$

where $\omega_{k,t}$ are the weights of the growth-optimal portfolio, held by logarithmic investors.

In comparison, the SDF of Chabi-Yo and Loudis (2020) is given by Equa-

tions (13) and (21) in their paper, as

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{1 + \sum_{k=1}^K \tilde{a}_{k,t} \tilde{R}_{M,t,T_1}^k}{1 + \sum_{k=1}^K \tilde{a}_{k,t} c_k} \quad (\text{I29})$$

with

$$\tilde{a}_{k,t} = \frac{a_{k,t}}{R_{f,t,T_1}^k}$$

The SDF (I29) can be expanded as

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{1}{1 + \sum_{k=1}^K \tilde{a}_{k,t} c_k} + \sum_{k=1}^K \left(\frac{\tilde{a}_{k,t}}{1 + \sum_{k=1}^K \tilde{a}_{k,t} c_k} \right) \tilde{R}_{M,t,T_1}^k \quad (\text{I30})$$

$$= 1 + \sum_{k=1}^K \left(\frac{\tilde{a}_{k,t}}{1 + \sum_{k=1}^K \tilde{a}_{k,t} c_k} \right) (\tilde{R}_{M,t,T_1}^k - c_k) \quad (\text{I31})$$

Setting the weights as follows:

$$\omega_{k,t} = R_{f,t,T_1} \frac{\tilde{a}_{k,t}}{1 + \sum_{k=1}^K \tilde{a}_{k,t} c_k}, \quad (\text{I32})$$

we recover the SDF of Tetlock, McCoy, and Shah (2024).

The model of Tetlock, McCoy, and Shah (2024) allows for investors who are not logarithmic investors. The complexity of these investors' portfolios determines the value of K . Consider $K = 2$, for illustration purposes. If the weights $\omega_{k,t}$ for $k = 1, \dots, K$ are known (or computed) as it is the case in Tetlock, McCoy, and Shah (2024), then the coefficients $\tilde{a}_{k,t}$ can be recovered from these weights and from the moments of market returns c_k . The preference parameters τ_t and ρ_t can be uniquely identified from these coefficients. Conversely, if the preference parameters τ_t and ρ_t are known, $\tilde{a}_{k,t}$ can be

recovered and the weights can be computed. The same reasoning extends to any finite $K > 2$.

G. Link to the model of Crescini, Trojani, and Vedolin (2025)

In this subsection, we show that the SDFs of Chabi-Yo and Loudis (2020) and Tetlock, McCoy, and Shah (2024) can be rewritten in the form of the SDF used by Crescini, Trojani, and Vedolin (2025).

From the spanning formula of Carr and Madan (2001), \tilde{R}_{M,t,T_1}^k , for $k > 1$ can be written as a collection of calls and puts:

$$\begin{aligned} \tilde{R}_{M,t,T_1}^k &= (R_{M,t,T_1} - R_{f,t,T_1})^k \\ &= \frac{k(k-1)}{S_t^2} \left\{ \int_{S_t R_{f,t,T_1}}^{\infty} \left(\frac{K}{S_t} - R_{f,t,T_1} \right)^{k-2} (S_{T_1} - K)^+ dK \right. \\ &\quad \left. + \int_0^{S_t R_{f,t,T_1}} \left(\frac{K}{S_t} - R_{f,t,T_1} \right)^{k-2} (K - S_{T_1})^+ dK \right\} \end{aligned} \quad (\text{I33})$$

where S_t is the spot market price at time t .

Substituting (I33) in (I28) and (I31), it follows that $\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}}$ is a weighted sum of the excess market return, and payoffs of calls and puts (both tradable) with maturity T_1 and strike K . This SDF can thus be rewritten in form of the SDF of Crescini, Trojani, and Vedolin (2025):

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = 1 + \theta' \mathbf{R}^e \quad (\text{I34})$$

where $\mathbf{R}^e = \mathbf{R} - 1$ contains the excess forward return of the index and options. $\theta' \mathbf{R}^e$ is a weighted sum of the excess market return and payoffs of tradable call and put options.

In the model of Crescini, Trojani, and Vedolin (2025), portfolio weights are allowed to vary across investors: $\theta = \theta_i$, to reflect varying demand of investors for the different assets. Logarithmic investors determine the SDF, which has the same form as the one of Tetlock, McCoy, and Shah (2024) and is a special case of Chabi-Yo and Loudis (2020). There is a mapping between the coefficients $a_{k,t}$ of Chabi-Yo and Loudis (2020) and the investors holdings in the vector θ of Equation (I34). Therefore, the coefficients $a_{k,t}$ embed information on aggregate demand for options with maturity T_1 .

H. Our inverse SDF as a portfolio of options

We show below that in our setting, the inverse SDF can be written as a portfolio of the market index and options with maturities T_1 and T_N .

Our inverse SDF can be viewed as an extension of the inverse SDFs of Chabi-Yo and Loudis (2020) in Equation (I29), augmented with the conditional risk-neutral variance of the market over $[T_1, T_N]$, $\mathbb{M}_{T_1, T_N}^{*(2)}$. We showed in Appendices I.F and I.G that the inverse SDF of Chabi-Yo and Loudis (2020) can be expressed as a weighted sum of the excess market return and a collection of option payoffs maturing at T_1 . As $\mathbb{M}_{T_1, T_N}^{*(k)} = \mathbb{E}_{T_1}^* \tilde{R}_{M, T_1, T_N}^k$ for $k > 1$ depends on future option prices, the spanning formula implies

$$\begin{aligned} \tilde{R}_{M, T_1, T_N}^k &= \left(\frac{S_{T_N}}{S_{T_1}} - R_{f, T_1, T_N} \right)^k \\ &= \frac{k(k-1)}{S_{T_1}^2} \left\{ \int_{S_{T_1} R_{f, T_1, T_N}}^{\infty} \left(\frac{K}{S_{T_1}} - R_{f, T_1, T_N} \right)^k (S_{T_N} - K)^+ dK \right. \\ &\quad \left. + \int_0^{S_{T_1} R_{f, T_1, T_N}} \left(\frac{K}{S_{T_1}} - R_{f, T_1, T_N} \right)^k (K - S_{T_N})^+ dK \right\} \end{aligned} \tag{I35}$$

Thus

$$\mathbb{M}_{T_1, T_N}^{*(k)} = \frac{k(k-1)R_{f, T_1, T_N}}{S_{T_1}^2} \left\{ \begin{array}{l} \int_{S_{T_1} R_{f, T_1, T_N}}^{\infty} \left(\frac{K}{S_{T_1}} - R_{f, T_1, T_N} \right)^k C_{T_N} [K] dK \\ + \int_0^{S_{T_1} R_{f, T_1, T_N}} \left(\frac{K}{S_{T_1}} - R_{f, T_1, T_N} \right)^k P_{T_N} [K] dK \end{array} \right\}, \quad (\text{I36})$$

where $C_{T_N}[K]$ and $P_{T_N}[K]$ are the prices of call and put options with strike K and maturity T_N . Therefore, our inverse SDF can be written as a weighted sum of the excess market return, a collection of payoffs on calls and puts and a set of payoffs from strategies that buy call and put options expiring at T_N and sell them at T_1 . These strategies arise due to intertemporal hedging in a multi-period economy. The loadings on these strategies, which depend on the coefficients $a_{k,t}$, embed information on aggregate demand for strategies that involve buying options with maturity T_N and selling them at T_1 .

II . Proofs and derivations of Section II

A. Contribution of intertemporal hedging in the model of Epstein and Zin (1989)

The structure imposed on the utility function, independent of the return process, is central to deriving the implications of intertemporal hedging for the equity risk premium. In this appendix, we detail how intertemporal hedging contributes to the equity risk premium when assuming Epstein and Zin (1989) utility.

Assuming that return on the wealth portfolio is equal to the market re-

turn, the SDF takes the form

$$m_{t,T_N} = \delta^\theta \left(\frac{C_{t,T_N}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{M,t,T_N}^{\theta-1}, \quad (\text{II37})$$

where δ is the discount factor, C_{t,T_N} is the consumption between times t and T_N , ψ is the elasticity of intertemporal substitution (EIS), and

$$\theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}}$$

with γ the risk aversion parameter.

Let us apply a Taylor-series expansion and use our methodology to demonstrate that intertemporal hedging directly contributes to the equity risk premium.

Consider the return of investing in asset k between t and T_1 , and in the risk-free asset between T_1 and T_N : $R_{k,t,T_1} R_{f,T_1,T_N}$. Under no-arbitrage conditions, the conditional expectation of this return can be expressed as

$$\begin{aligned} \mathbb{E}_t(R_{k,t,T_1} R_{f,T_1,T_N}) &= \mathbb{E}_t \left(\frac{m_{t,T_N}}{\mathbb{E}_t m_{t,T_N}} \frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} R_{k,t,T_1} R_{f,T_1,T_N} \right) = \mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} R_{k,t,T_1} R_{f,T_1,T_N} \right) \\ &= \mathbb{E}_t^* \left(R_{f,T_1,T_N} R_{k,t,T_1} \mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) \right) \\ &= R_{f,T_1,T_N} \mathbb{E}_t^* \left(R_{k,t,T_1} \mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) \right) \end{aligned} \quad (\text{II38})$$

Therefore we have

$$\mathbb{E}_t(R_{k,t,T_1}) = \mathbb{E}_t^* \left(R_{k,t,T_1} \frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \right) = \mathbb{E}_t^* \left(R_{k,t,T_1} \mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) \right). \quad (\text{II39})$$

The uniqueness of the SDF implies that:

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) \quad (\text{II40})$$

Let us now calculate the right handside of Equation (II40) using the expression of the Epstein-Zin SDF:

$$m_{t,T_N} = \delta^\theta \left(\frac{C_{t,T_1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{-\frac{\theta}{\psi}} R_{M,t,T_1}^{\theta-1} R_{M,T_1,T_N}^{\theta-1}. \quad (\text{II41})$$

Thus,

$$\frac{1}{m_{t,T_N}} = \delta^{-\theta} \left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta}, \quad (\text{II42})$$

and

$$\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} = \frac{1}{R_{f,t,T_N}} \delta^{-\theta} \left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta}. \quad (\text{II43})$$

Using $\mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) = 1$ yields

$$\mathbb{E}_t^* \left(\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta} \right) = \delta^\theta R_{f,t,T_N} \quad (\text{II44})$$

and hence

$$\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} = \frac{\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta}}{\mathbb{E}_t^* \left(\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta} \right)}. \quad (\text{II45})$$

We then take the expectation at T_1 :

$$\mathbb{E}_{T_1}^* \left(\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} \right) = \frac{\mathbb{E}_{T_1}^* \left(\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta} \right)}{\mathbb{E}_t^* \left(\mathbb{E}_{T_1}^* \left(\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta} \right) \right)}. \quad (\text{II46})$$

Now, we perform an expansion series of $\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta}$ around²

$$\left(\frac{C_{t,T_1}}{C_t}, \frac{C_{T_1,T_N}}{C_{t,T_1}}, R_{M,t,T_1}, R_{M,T_1,T_N} \right) = (1, 1, R_{f,t,T_1}, R_{f,T_1,T_N}). \quad (\text{II47})$$

Taking the T_1 - conditional expectation of this expansion-series produces

$$\begin{aligned} & \mathbb{E}_{T_1}^* \left(\left(\frac{C_{t,T_1}}{C_t} \right)^{\frac{\theta}{\psi}} \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} \right)^{\frac{\theta}{\psi}} R_{M,t,T_1}^{1-\theta} R_{M,T_1,T_N}^{1-\theta} \right) \\ &= A_0 + A_1 \left(\frac{C_{t,T_1}}{C_t} - 1 \right) + A_2 \left(\mathbb{E}_{T_1}^* \frac{C_{T_1,T_N}}{C_{t,T_1}} - 1 \right) + A_3 (R_{M,t,T_1} - R_{f,t,T_1}) \\ &+ A_4 \left(\frac{C_{t,T_1}}{C_t} - 1 \right) (R_{M,t,T_1} - R_{f,t,T_1}) + A_5 \left(\frac{C_{t,T_1}}{C_t} - 1 \right) \left(\mathbb{E}_{T_1}^* \frac{C_{T_1,T_N}}{C_{t,T_1}} - 1 \right) \\ &+ A_6 \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} - 1 \right) (R_{M,t,T_1} - R_{f,t,T_1}) + A_7 \mathbb{E}_{T_1}^* \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} - 1 \right) (R_{M,T_1,T_N} - R_{f,T_1,T_N}) \\ &+ A_8 \left(\frac{C_{t,T_1}}{C_t} - 1 \right)^2 + A_9 \mathbb{E}_{T_1}^* \left(\frac{C_{T_1,T_N}}{C_{t,T_1}} - 1 \right)^2 + A_{10} (R_{M,t,T_1} - R_{f,t,T_1})^2 \\ &+ A_{11} \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N})^2 \end{aligned} \quad (\text{II48})$$

²An expansion series around $\left(\mathbb{E}_t^* \frac{C_{t,T_1}}{C_t}, \mathbb{E}_{T_1}^* \frac{C_{T_1,T_N}}{C_{t,T_1}}, R_{f,t,T_1}, R_{f,T_1,T_N} \right)$ has similar implications.

with

$$\begin{aligned}
A_0 &= R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, & A_1 &= A_2 = \frac{\theta}{\psi} R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, \\
A_3 &= (1-\theta) R_{f,t,T_1}^{-\theta} R_{f,T_1,T_N}^{1-\theta}, & A_4 &= A_6 = \frac{\theta(1-\theta)}{\psi} R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, \\
A_5 &= \left(\frac{\theta}{\psi}\right)^2 R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, & A_7 &= \frac{\theta(1-\theta)}{\psi} R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, \\
A_8 &= A_9 = \frac{1}{2} \left(\frac{\theta}{\psi}\right) \left(\frac{\theta}{\psi} - 1\right) R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, \\
A_{10} &= \frac{1}{2} (\theta - 1) \theta R_{f,t,T_1}^{-(\theta+1)} R_{f,T_1,T_N}^{1-\theta}, & A_{11} &= \frac{1}{2} (\theta - 1) \theta R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{-(\theta+1)}
\end{aligned} \tag{II49}$$

Hence the inverse SDF takes the form

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{Z_{T_1}}{\mathbb{E}_t^* Z_{T_1}} \tag{II50}$$

with

$$\begin{aligned}
Z_{T_1} &= 1 + (A_1/A_0) \left(\frac{C_{t,T_1}}{C_t} - 1\right) + (A_2/A_0) \left(\mathbb{E}_{T_1}^* \frac{C_{T_1,T_N}}{C_{T_1}} - 1\right) + (A_3/A_0) (R_{M,t,T_1} - R_{f,t,T_1}) \\
&+ (A_4/A_0) \left(\frac{C_{t,T_1}}{C_t} - 1\right) (R_{M,t,T_1} - R_{f,t,T_1}) + (A_5/A_0) \left(\frac{C_{t,T_1}}{C_t} - 1\right) \left(\mathbb{E}_{T_1}^* \frac{C_{T_1,T_N}}{C_{T_1}} - 1\right) \\
&+ (A_6/A_0) (R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{E}_{T_1}^* \left(\frac{C_{T_1,T_N}}{C_{T_1}} - 1\right) \\
&+ (A_7/A_0) \mathbb{E}_{T_1}^* \left(\frac{C_{T_1,T_N}}{C_{T_1}} - 1\right) (R_{M,T_1,T_N} - R_{f,T_1,T_N}) + (A_8/A_0) \left(\frac{C_{t,T_1}}{C_t} - 1\right)^2 \\
&+ (A_9/A_0) \mathbb{E}_{T_1}^* \left(\frac{C_{T_1,T_N}}{C_{T_1}} - 1\right)^2 + (A_{10}/A_0) (R_{M,t,T_1} - R_{f,t,T_1})^2 \\
&+ (A_{11}/A_0) \text{VAR}_{T_1}^* (R_{M,T_1,T_N}).
\end{aligned} \tag{II51}$$

The Epstein–Zin formulation of the SDF (II41) directly produces an SDF

expressed as a function of market returns and consumption growth, without volatility playing any direct role. To highlight the impact of volatility on the SDF, a structure is often imposed on the return process. In contrast, in our approach, we do not impose any constraint on the return process. A Taylor-series expansion of marginal utility makes it transparent how market volatility affects the SDF, through the term $(A_{10}/A_0) (R_{M,t,T_1} - R_{f,t,T_1})^2$. Similarly, market volatility after T_1 affects the SDF through the term $(A_{11}/A_0) \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* (R_{M,T_1,T_N})$.

Using this inverse SDF and applying (I2), the expected excess return on the market is

$$\mathbb{E} (R_{M,t,T_1} - R_{f,t,T_1}) = \frac{\mathbb{C}\mathbb{O}\mathbb{V}_t^* (R_{M,t,T_1}, Z_{T_1})}{\mathbb{E}_t^* Z_{T_1}}. \quad (\text{II52})$$

The effect of intertemporal hedging on the equity risk premium happens through the term $\mathbb{C}\mathbb{O}\mathbb{V}_t^* (R_{M,t,T_1}, \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* (R_{M,T_1,T_N}))$. This effect is given by

$$\frac{A_{11}}{A_0} \frac{\mathbb{C}\mathbb{O}\mathbb{V}_t^* (R_{M,t,T_1}, \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* (R_{M,T_1,T_N}))}{\mathbb{E}_t^* Z_{T_1}}, \quad (\text{II53})$$

with

$$\frac{A_{11}}{A_0} = \frac{1(\theta - 1)\theta}{2R_{f,T_1,T_N}^2} = \frac{1}{2} \frac{1}{R_{f,T_1,T_N}^2} \frac{\psi\gamma(\gamma - 1) \left(\psi - \frac{1}{\gamma}\right)}{(\psi - 1)^2} \quad (\text{II54})$$

When θ is between 0 and 1, $(\theta - 1)\theta < 0$, and intertemporal hedging increases the equity risk premium through (II53) if $\mathbb{C}\mathbb{O}\mathbb{V}_t^* (R_{M,t,T_1}, \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* (R_{M,T_1,T_N})) < 0$.

In contrast, when $(\theta - 1)\theta > 0$, intertemporal hedging decreases the equity risk premium through (II53) if $\mathbb{C}\mathbb{O}\mathbb{V}_t^* (R_{M,t,T_1}, \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* (R_{M,T_1,T_N})) < 0$. This result holds true, in particular, when $\gamma > 1$ and $\psi > 1$, which implies

that θ is negative and $A_{11} > 0$.

B. Contribution of intertemporal hedging in the model of Campbell and Cochrane (1999)

In the model of Campbell and Cochrane (1999) with external habit formation, consumption and wealth growth are linked through the surplus ratio and the return on aggregate wealth:

$$\frac{C_{t,T_N}}{C_t} = \frac{s_{t,T_N}}{s_t} R_{M,t,T_N} \quad (\text{II55})$$

where s_t is the surplus consumption ratio at time t , and s_{t,T_N} is the change in surplus consumption ratio between t and T_N (habit effect).

Plugging this relation into the Epstein-Zin SDF links the SDF directly to market returns and habit dynamics:

$$m_{t,T_N} = \delta^\theta \left(\frac{s_{t,T_N}}{s_t} \right)^{-\frac{\theta}{\psi}} R_{M,t,T_N}^{\theta-1-\frac{\theta}{\psi}}, \quad (\text{II56})$$

Campbell and Cochrane (1999) show that this representation of the SDF produces countercyclical risk aversion and time-varying volatility premia consistent with the data.

Using the same steps as in Appendix Section II.A produces

$$\begin{aligned}
Z_{T_1} = & 1 + (B_1/A_0) \left(\frac{s_{t,T_1}}{s_t} - 1 \right) + (B_2/A_0) \left(\mathbb{E}_{T_1}^* \frac{s_{T_1,T_N}}{s_{T_1}} - 1 \right) \\
& + (B_3/A_0) (R_{M,t,T_1} - R_{f,t,T_1}) + (B_4/A_0) \left(\frac{s_{t,T_1}}{s_t} - 1 \right) (R_{M,t,T_1} - R_{f,t,T_1}) \\
& + (B_5/A_0) \left(\frac{s_{t,T_1}}{s_t} - 1 \right) \left(\mathbb{E}_{T_1}^* \frac{s_{T_1,T_N}}{s_{T_1}} - 1 \right) + (B_6/A_0) (R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{E}_{T_1}^* \left(\frac{s_{T_1,T_N}}{s_{T_1}} - 1 \right) \\
& + (B_7/A_0) \mathbb{E}_{T_1}^* \left(\frac{s_{T_1,T_N}}{s_{T_1}} - 1 \right) (R_{M,T_1,T_N} - R_{f,T_1,T_N}) + (B_8/A_0) \left(\frac{s_{t,T_1}}{s_t} - 1 \right)^2 \\
& + (B_9/A_0) \mathbb{E}_{T_1}^* \left(\frac{s_{T_1,T_N}}{s_{T_1}} - 1 \right)^2 + (B_{10}/A_0) (R_{M,t,T_1} - R_{f,t,T_1})^2 \\
& + (B_{11}/A_0) \text{VAR}_{T_1}^* (R_{M,T_1,T_N}). \tag{II57}
\end{aligned}$$

with

$$\begin{aligned}
B_0 &= R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, & B_1 &= B_2 = \frac{\theta}{\psi} R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, \\
B_3 &= \left(1 - \theta + \frac{\theta}{\psi} \right) R_{f,t,T_1}^{-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, & B_4 &= B_7 = \frac{\theta \left(1 - \theta + \frac{\theta}{\psi} \right)}{\psi} R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, \\
B_5 &= \left(\frac{\theta}{\psi} \right)^2 R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, & B_6 &= \frac{\theta \left(1 - \theta + \frac{\theta}{\psi} \right)}{\psi} R_{f,t,T_1}^{1-\theta} R_{f,T_1,T_N}^{1-\theta}, \\
B_8 &= B_9 = \frac{1}{2} \left(\frac{\theta}{\psi} \right) \left(\frac{\theta}{\psi} - 1 \right) R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{1-\theta+\frac{\theta}{\psi}}, & B_{10} &= \frac{1}{2} \left(1 - \theta + \frac{\theta}{\psi} \right) \left(-\theta + \frac{\theta}{\psi} \right) R_{f,t,T_1}^{-\theta+\frac{\theta}{\psi}-1}, \\
B_{11} &= \frac{1}{2} \left(1 - \theta + \frac{\theta}{\psi} \right) \left(-\theta + \frac{\theta}{\psi} \right) R_{f,t,T_1}^{1-\theta+\frac{\theta}{\psi}} R_{f,T_1,T_N}^{-\theta+\frac{\theta}{\psi}-1}. \tag{II58}
\end{aligned}$$

The contribution of intertemporal hedging to the equity risk premium happens through the term $\text{COV}_t^* (R_{M,t,T_1}, \text{VAR}_{T_1}^* (R_{M,T_1,T_N}))$. This contribution is given by

$$\frac{B_{11}}{A_0} \frac{\text{COV}_t^* (R_{M,t,T_1}, \text{VAR}_{T_1}^* (R_{M,T_1,T_N}))}{\mathbb{E}_t^* Z_{T_1}}, \tag{II59}$$

with

$$\frac{B_{11}}{A_0} = \frac{1}{2} \frac{\left(\theta - \frac{\theta}{\psi} - 1\right) \left(\theta - \frac{\theta}{\psi}\right)}{R_{f,T_1,T_N}^2} = \frac{1}{2} \frac{\gamma(\gamma - 1)}{R_{f,T_1,T_N}^2}$$

Interestingly, the sign of $\frac{B_{11}}{A_0}$ does not depend on ψ , which simplifies out of the equation. Hence, in this model, assuming that the leverage effect $\text{COV}_t^*(R_{M,t,T_1}, \text{VAR}_{T_1}^*(R_{M,T_1,T_N})) < 0$, if $\gamma > 1$, then $\gamma(\gamma - 1) > 0$ and hence intertemporal hedging decreases the equity risk premium through (II59). If, in turn, $\gamma < 1$, then intertemporal hedging increases the equity risk premium.

C. Recursive utility with a general aggregator

In this section, we use a Taylor-series expansion to characterize the contribution of intertemporal hedging to the total equity risk premium without imposing a specific functional form on the aggregator and on the certainty equivalent, as it is done by Epstein and Zin (1989, 1991). We show that the direction of the relationship between intertemporal hedging and the equity risk premium is dictated by the curvature properties of the aggregator and certainty-equivalent functions. When both take a power form, the shape of these functions is determined by a single parameter, and intertemporal hedging reduces the equity risk premium under common preference parameter calibrations. Epstein and Zin used power forms for tractability.

Plugging the functional forms (20) in (18) and (19), the Epstein–Zin aggregator is given by

$$U_t = \left((1 - \beta) C_t^{1 - \frac{1}{\psi}} + \beta (\mu_t [U_{t+1}])^{1 - \frac{1}{\psi}} \right)^{\frac{1}{1 - \frac{1}{\psi}}} \quad (\text{II60})$$

and the certainty equivalent $\mu_t [U_{t+1}]$ is

$$\mu_t [U_{t+1}] = (\mathbb{E}_t [U_{t+1}^{1-\gamma}])^{\frac{1}{1-\gamma}}. \quad (\text{II61})$$

Both the certainty equivalent $\mu_t [U_{t+1}]$ and the aggregator U_t take a Constant Elasticity of Substitution (CES) form, similar to the CRRA specification.

Let us now consider the more general recursive preferences of Equations (18)-(19), and assume that the functional forms of $\vartheta [\cdot]$, $g [\cdot]$, and $v [\cdot]$ are not known and $\vartheta [\cdot] \neq v [\cdot]$. Assuming regularity conditions on $\vartheta [\cdot]$, $g [\cdot]$, and $v [\cdot]$, the SDF over the period $[t, T_N]$ is

$$\begin{aligned} m_{t,T_N} &\propto \beta \frac{g' [C_{T_N}]}{g' [C_t]} \frac{\vartheta' [U_t]}{\vartheta' [U_{T_N}]} \frac{v' [U_{T_N}]}{v' [\mu_t]} \\ &\propto \beta \frac{g' [C_{T_N}]}{g' [C_t]} \frac{\vartheta' [\mu_t]}{\vartheta' [U_{T_N}]} \frac{v' [U_{T_N}]}{v' [\mu_t]} = \frac{const}{Z_{T_N}} \end{aligned} \quad (\text{II62})$$

with

$$Z_{T_N} = \frac{g' [C_t]}{g' [C_{T_N}]} \frac{\vartheta' [U_{T_N}]}{\vartheta' [\mu_t]} \frac{v' [\mu_t]}{v' [U_{T_N}]} \quad (\text{II63})$$

From Equation (II62) we have

$$const = m_{t,T_N} Z_{T_N} = \mathbb{E}_t m_{t,T_N} Z_{T_N} = \mathbb{E}_t m_{t,T_N} \mathbb{E}_t^* Z_{T_N} \quad (\text{II64})$$

Hence

$$\frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} = \frac{Z_{T_N}}{\mathbb{E}_t^* Z_{T_N}}$$

Now, use Equation (II40) to infer

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \mathbb{E}_{T_1}^* \frac{\mathbb{E}_t m_{t,T_N}}{m_{t,T_N}} = \frac{\mathbb{E}_{T_1}^* Z_{T_N}}{\mathbb{E}_t^* Z_{T_N}}. \quad (\text{II65})$$

Z_{T_N} depends on C_{T_N} and U_{T_N} . U_{T_N} , in turn, depends on the market return: $U_{T_N} = U_{T_N}[W_{T_N}]$ with $W_{T_N} = (W_t - C_t) R_{M,t,T_N}$. A second order Taylor expansion-series of $\frac{U_{T_N}[W_{T_N}]}{\mu_t}$ around $w_0 = (W_t - C_t) R_{f,t,T_N}$ produces

$$\frac{U_{T_N}}{\mu_t} = \underbrace{\frac{U_{T_N}[w_0]}{\mu_t}}_{\lambda_{0,t}} + \underbrace{\frac{(W_t - C_t) U'_{T_N}[w_0]}{\mu_t}}_{\lambda_{1,t}} (R_{M,t,T_N} - R_{f,t,T_N}) + \underbrace{\frac{(W_t - C_t)^2 U''_{T_N}[w_0]}{\mu_t}}_{\lambda_{2,t}} (R_{M,t,T_N} - R_{f,t,T_N})^2 \quad (\text{II66})$$

which simplifies to

$$\frac{U_{T_N}}{\mu_t} = 1 + \lambda_{1,t}^* (R_{M,t,T_N} - R_{f,t,T_N}^*) + \lambda_{2,t} (R_{M,t,T_N} - R_{f,t,T_N}^*)^2 \quad (\text{II67})$$

where R_{f,t,T_N}^* and $\lambda_{1,t}^*$ are chosen so that Equations (II66) and (II67) match:³

$$\begin{aligned} \lambda_{1,t}^* &= \lambda_{1,t} - 2\lambda_{2,t}(R_{f,t,T_N} - R_{f,t,T_N}^*) \quad \text{and} \\ R_{f,t,T_N}^* &= R_{f,t,T_N} - \frac{\lambda_{1,t} \pm \sqrt{\lambda_{1,t}^2 - 4\lambda_{2,t}(\lambda_{0,t} - 1)}}{2\lambda_{2,t}}. \end{aligned} \quad (\text{II68})$$

Now that we have rewritten U_{T_N} as a function of $R_{M,t,T_N} = R_{M,t,T_1} R_{M,T_1,T_N}$, as a result, Z_{T_N} depends on $(C_{T_N}, R_{M,t,T_1}, R_{M,T_1,T_N})$. A second-order Taylor

³We assume that (II66) can be rewritten into (II67) for tractability. The matching can be done if $\lambda_{1,t}^2 - 4\lambda_{2,t}(\lambda_{0,t} - 1) \geq 0$, $\lambda_{2,t} < 0$ and the resulting $R_{f,t,T_N}^* \geq 0$. If these conditions are not satisfied, the calculations are more cumbersome but the same reasoning holds.

expansion-series of Z_{T_N} around $(C_{T_N}, R_{M,t,T_1}, R_{M,T_1,T_N}) = (C_t, R_{f,t,T_1}^*, R_{f,T_1,T_N}^*)$ yields

$$\begin{aligned} Z_{T_N} &= 1 + H + \lambda (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2 \\ &= 1 + H + \lambda (R_{M,T_1,T_N} - R_{f,T_1,T_N} + R_{f,T_1,T_N} - R_{f,T_1,T_N}^*)^2 \quad (\text{II69}) \end{aligned}$$

where H contains all the terms of the expansion-series except $\lambda (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2$.

Since

$$\mathbb{E}_{T_1}^* Z_{T_N} = 1 + \lambda (R_{f,T_1,T_N} - R_{f,T_1,T_N}^*)^2 + \mathbb{E}_{T_1}^* H + \lambda \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2, \quad (\text{II70})$$

we have

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} = \frac{\left\{ 1 + \lambda (R_{f,T_1,T_N} - R_{f,T_1,T_N}^*)^2 + \mathbb{E}_{T_1}^* H + \lambda \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2 \right\}}{\left\{ \begin{array}{c} 1 + \lambda (R_{f,T_1,T_N} - R_{f,T_1,T_N}^*)^2 \\ + \mathbb{E}_t^* (\mathbb{E}_{T_1}^* H) + \lambda \mathbb{E}_t^* (\mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2) \end{array} \right\}}$$

Our goal is to determine λ because the contribution of leverage to the equity risk premium is $\lambda \text{COV}_t^* (R_{M,t,T_1}, \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N}^*)^2)$. Let us set $(x, y) = (R_{M,t,T_1}, R_{M,T_1,T_N})$ and $(x_0^*, y_0^*) = (R_{f,t,T_1}^*, R_{f,T_1,T_N}^*)$.

$$\begin{aligned}
\lambda &= \frac{1}{2} \left\{ \frac{\partial^2 \left(\frac{\vartheta' [U_{TN}]}{\vartheta' [\mu_t]} \right)}{\partial^2 R_{M,T_1,T_N}} + 2 \frac{\partial \left(\frac{\vartheta' [U_{TN}]}{\vartheta' [\mu_t]} \right)}{\partial R_{M,T_1,T_N}} \frac{\partial \left(\frac{v' [\mu_t]}{v' [U_{TN}]} \right)}{\partial R_{M,T_1,T_N}} + \frac{\partial^2 \left(\frac{v' [\mu_t]}{v' [U_{TN}]} \right)}{\partial^2 R_{M,T_1,T_N}} \right\}_{(x,y)=(x_0^*,y_0^*)} \\
& \quad \text{(II71)} \\
&= \frac{1}{2} \left\{ \begin{aligned} & \frac{\vartheta''' [U_{TN}]}{\vartheta' [\mu_t]} (U'_{TN})^2 + \frac{\vartheta'' [U_{TN}]}{\vartheta' [\mu_t]} U''_{TN} - 2 \frac{\vartheta'' [U_{TN}]}{\vartheta' [\mu_t]} \frac{v'' [U_{TN}] v' [\mu_t]}{(v' [U_{TN}])^2} (U'_{TN})^2 \\ & - \frac{v''' [U_{TN}] v' [\mu_t]}{(v' [U_{TN}])^2} (U'_{TN})^2 + 2 \frac{(v'' [U_{TN}])^2 v' [\mu_t]}{(v' [U_{TN}])^3} (U'_{TN})^2 - \frac{v'' [U_{TN}] v' [\mu_t]}{(v' [U_{TN}])^2} U''_{TN} \end{aligned} \right\}_{(x,y)=(x_0^*,y_0^*)} \\
&= \frac{1}{2} \left\{ \left(\begin{aligned} & \frac{\vartheta''' [\mu_t]}{\vartheta' [\mu_t]} - 2 \frac{\vartheta'' [\mu_t] v'' [\mu_t]}{\vartheta' [\mu_t] v' [\mu_t]^2} \\ & - \frac{v''' [U_{TN}]}{v' [\mu_t]} + 2 \frac{(v'' [\mu_t])^2}{(v' [\mu_t])^2} \end{aligned} \right) \mu_t^2 \lambda_{1,t}^{*2} + \left(\frac{\vartheta'' [\mu_t]}{\vartheta' [\mu_t]} - \frac{v'' [\mu_t]}{v' [\mu_t]} \right) 2 \mu_t \lambda_{2,t} \right\}_{(x,y)=(x_0^*,y_0^*)} \\
&= \left(\frac{\rho_t^\vartheta}{(\tau_t^\vartheta)^2} - \frac{1}{\tau_t^v} \frac{1}{\tau_t^\vartheta} + \frac{(1 - \rho_t^v)}{(\tau_t^v)^2} \right) \lambda_{1,t}^{*2} + \left(\frac{1}{\tau_t^v} - \frac{1}{\tau_t^\vartheta} \right) \lambda_{2,t} \\
&= \left(\left(\frac{1}{\tau_t^\vartheta} - \frac{1}{\tau_t^v} \right) \frac{1}{\tau_t^\vartheta} + \frac{(1 - \rho_t^v)}{(\tau_t^v)^2} - \frac{(1 - \rho_t^\vartheta)}{(\tau_t^\vartheta)^2} \right) \lambda_{1,t}^{*2} + \left(\frac{1}{\tau_t^v} - \frac{1}{\tau_t^\vartheta} \right) \lambda_{2,t} \\
& \quad \text{(II72)}
\end{aligned}$$

where

$$\frac{1}{\tau_t^\vartheta} = -\frac{\mu_t \vartheta'' [\mu_t]}{\vartheta' [\mu_t]}, \quad \rho_t^\vartheta = \frac{1}{2} \frac{\vartheta' [\mu_t] \vartheta''' [\mu_t]}{(\vartheta'' [\mu_t])^2}, \quad \frac{1}{\tau_t^v} = -\frac{\mu_t v'' [\mu_t]}{v' [\mu_t]}, \quad \rho_t^v = \frac{1}{2} \frac{v' [\mu_t] v''' [\mu_t]}{(v'' [\mu_t])^2}$$

The sign of λ is therefore determined by the four preference parameters τ_t^ϑ , τ_t^v , ρ_t^ϑ and ρ_t^v , which capture the shape of the aggregator and the certainty equivalent. λ can be either positive or negative, depending on these four parameters.

In the case of Epstein-Zin utility,

$$\frac{U_{T_N}}{\mu_t} = (R_{M,t,T_N})^{\xi(\gamma,\psi)} \text{ with } \xi(\gamma,\psi) = \frac{1 - \frac{1}{\psi} - \gamma}{\frac{1}{\psi} \left(\frac{1}{\psi} - \gamma \right)} \text{ and}$$

$$\frac{1}{\tau_t^\vartheta} = \frac{1}{\psi}, \frac{1}{\tau_t^v} = \gamma, \rho_t^\vartheta = \frac{\frac{1}{\psi} + 1}{\frac{1}{\psi}} \text{ and } \rho_t^v = \frac{\gamma + 1}{\gamma}.$$

This implies that λ only depends on two parameters, γ and ψ :

$$\lambda = \left(\frac{1}{2\psi} \left(\frac{1}{\psi} + 1 \right) - \frac{\gamma}{\psi} + \gamma^2 \left(1 - \frac{1}{2} \frac{(\gamma + 1)}{\gamma} \right) \right) \xi(\gamma,\psi)^2 + \left(\gamma - \frac{1}{\psi} \right) \xi(\gamma,\psi) (\xi(\gamma,\psi) - 1)$$

(II73)

Using standard calibrations yields

$$\gamma = 10, \psi = 2 \text{ and } \lambda \approx 180.5$$

$$\gamma = 10, \psi = 1 \text{ and } \lambda \approx 45.56$$

$$\gamma = 10, \psi = 3 \text{ and } \lambda \approx 404.55.$$

Since $\lambda > 0$ in these calibrations, under Epstein Zin preferences, intertemporal hedging decreases the equity risk premium. But in the general case, λ as given by Equation (II72) is not constrained to be positive.

D. Recursive utility in the case of small departures from Epstein-Zin utility

Assuming that regularity conditions are satisfied on $g[\cdot]$, $\vartheta[\cdot]$ and $v[\cdot]$, we now consider the case in which these functions are first-order departures

from the power forms used in Epstein Zin utility. Let us denote

$$g[x] = f_g[g_{EZ}[x]], \vartheta[x] = f_\vartheta[\vartheta_{EZ}[x]], v[x] = f_v[v_{EZ}[x]] \quad (\text{II74})$$

where ϑ_{EZ} , g_{EZ} and v_{EZ} are the function forms of Epstein Zin preferences, given in Equation (20). Since $\vartheta_{EZ}[1] = 1$, $g_{EZ}[1] = 1$ and $v_{EZ}[1] = 1$, we set $g[1] = 1$, $\vartheta[1] = 1$ and $v[1] = 1$ as well for simplicity. Now let us revisit the coefficient λ derived in Equation (II71), which captures the contribution of the leverage term to the equity risk premium. Notice that

$$\begin{aligned} \frac{\partial \left(\frac{\vartheta'[U_{T_N}]}{\vartheta'[\mu_t]} \right)}{\partial R_{M,T_1,T_N}} &= \frac{\partial \left(\frac{f'_\vartheta[\vartheta_{EZ}[U_{T_N}]]\vartheta'_{EZ}[U_{T_N}]}{f'_\vartheta[\vartheta_{EZ}[\mu_t]]\vartheta'_{EZ}[\mu_t]} \right)}{\partial R_{M,T_1,T_N}} \\ &= \frac{f''_\vartheta[\vartheta_{EZ}[U_{T_N}]](\vartheta'_{EZ}[U_{T_N}])^2}{f'_\vartheta[\vartheta_{EZ}[\mu_t]]\vartheta'_{EZ}[\mu_t]} U'_{T_N} + \frac{f'_\vartheta[\vartheta_{EZ}[U_{T_N}]]\vartheta''_{EZ}[U_{T_N}]}{f'_\vartheta[\vartheta_{EZ}[\mu_t]]\vartheta'_{EZ}[\mu_t]} U'_{T_N}. \end{aligned}$$

Hence,

$$\begin{aligned} \left(\frac{\partial \left(\frac{\vartheta'[U_{T_N}]}{\vartheta'[\mu_t]} \right)}{\partial R_{M,T_1,T_N}} \right)_{(x,y)=(x_0^*,y_0^*)} &= \frac{\vartheta_{EZ}[\mu_t] f''_\vartheta[\vartheta_{EZ}[\mu_t]] \vartheta'_{EZ}[\mu_t] \mu_t}{f'_\vartheta[\vartheta_{EZ}[\mu_t]] \vartheta_{EZ}[\mu_t]} \lambda_{1,t}^* + \frac{\mu_t \vartheta''_{EZ}[\mu_t]}{\vartheta'_{EZ}[\mu_t]} \lambda_{1,t}^* \\ &= -\frac{1}{\tau_t^{f_\vartheta}} \left(1 - \frac{1}{\psi}\right) \lambda_{1,t}^* - \frac{1}{\psi} \lambda_{1,t}^* \end{aligned} \quad (\text{II75})$$

where

$$\frac{1}{\tau_t^{f_\vartheta}} = -\frac{\vartheta_{EZ}[\mu_t] f''_\vartheta[\vartheta_{EZ}[\mu_t]]}{f'_\vartheta[\vartheta_{EZ}[\mu_t]]}. \quad (\text{II76})$$

Furthermore,

$$\frac{\partial^2 \left(\frac{\vartheta' [U_{T_N}]}{\vartheta' [\mu_t]} \right)}{\partial^2 R_{M,T_1,T_N}} = \frac{\left\{ \begin{aligned} & \left[f_{\vartheta}''' [\vartheta_{EZ} [U_{T_N}]] [\vartheta'_{EZ} [U_{T_N}]]^3 + 3 f_{\vartheta}'' [\vartheta_{EZ} [U_{T_N}]] \vartheta'_{EZ} [U_{T_N}] \vartheta''_{EZ} [U_{T_N}] \right. \\ & \quad \left. + f_{\vartheta}' [\vartheta_{EZ} [U_{T_N}]] \vartheta'''_{EZ} [U_{T_N}] \right] (U'_{T_N})^2 \\ & \quad \left. + \left[f_{\vartheta}'' [\vartheta_{EZ} [U_{T_N}]] (\vartheta'_{EZ} [U_{T_N}])^2 + f_{\vartheta}' [\vartheta_{EZ} [U_{T_N}]] \vartheta''_{EZ} [U_{T_N}] \right] U''_{T_N} \right\}}{f_{\vartheta}' [\vartheta_{EZ} [\mu_t]] \vartheta'_{EZ} [\mu_t]} \quad (\text{II77}) \end{aligned}$$

Hence,

$$\begin{aligned} \left(\frac{\partial^2 \left(\frac{\vartheta' [U_{T_N}]}{\vartheta' [\mu_t]} \right)}{\partial^2 R_{M,T_1,T_N}} \right)_{(x,y)=(x_0^*,y_0^*)} &= \left\{ \frac{f_{\vartheta}''' [\vartheta_{EZ} [\mu_t]] (\vartheta'_{EZ} [\mu_t])^2}{f_{\vartheta}' [\vartheta_{EZ} [\mu_t]]} \mu_t^2 + \frac{3 f_{\vartheta}'' [\vartheta_{EZ} [\mu_t]] \vartheta''_{EZ} [\mu_t]}{f_{\vartheta}' [\vartheta_{EZ} [\mu_t]]} \mu_t^2 \right\} (\lambda_{1,t}^*)^2 \\ &+ 2 \mu_t \left[\frac{f_{\vartheta}'' (\vartheta_{EZ} [\mu_t])}{f_{\vartheta}' (\vartheta_{EZ} [\mu_t])} \vartheta'_{EZ} [\mu_t] + \frac{\vartheta''_{EZ} [\mu_t]}{\vartheta'_{EZ} [\mu_t]} \right] \lambda_{2,t} + \frac{\vartheta'''_{EZ} [\mu_t] \mu_t^2}{\vartheta'_{EZ} [\mu_t]} (\lambda_{1,t}^*)^2 \\ &= \left\{ \begin{aligned} & \frac{2(1-\frac{1}{\psi})^2 \rho_t^{f_{\vartheta}}}{(\tau_t^{f_{\vartheta}})^2} - \frac{3}{\psi} \left(1 - \frac{1}{\psi}\right) \frac{1}{\tau_t^{f_{\vartheta}}} \\ & \quad + \frac{1}{\psi} \left(1 + \frac{1}{\psi}\right) \end{aligned} \right\} (\lambda_{1,t}^*)^2 \\ &- \left[\frac{2}{\psi} + 2 \left(1 - \frac{1}{\psi}\right) \frac{1}{\tau_t^{f_{\vartheta}}} \right] \lambda_{2,t} \quad (\text{II78}) \end{aligned}$$

with

$$\rho_t^{f_{\vartheta}} = \frac{1}{2} \frac{f_{\vartheta}''' [\vartheta_{EZ} [\mu_t]] f_{\vartheta}' [\vartheta_{EZ} [\mu_t]]}{(f_{\vartheta}'' [\vartheta_{EZ} [\mu_t]])^2}. \quad (\text{II79})$$

Similarly,

$$\begin{aligned} \frac{\partial \left(\frac{v'[\mu_t]}{v'[U_{T_N}]} \right)}{\partial R_{M,T_1,T_N}} &= \frac{\partial \left(\frac{f'_v[v_{EZ}[\mu_t]]v'_{EZ}[\mu_t]}{f'_v[v_{EZ}[U_{T_N}]]v'_{EZ}[U_{T_N}]} \right)}{\partial R_{M,T_1,T_N}} \\ &= - \frac{f'_v[v_{EZ}[\mu_t]]v'_{EZ}[\mu_t] \left(f''_v[v_{EZ}[U_{T_N}]](v'_{EZ}[U_{T_N}])^2 U'_{T_N} + f'_v[v_{EZ}[U_{T_N}]]v''_{EZ}[U_{T_N}]U'_{T_N} \right)}{(f'_v[v_{EZ}[U_{T_N}]]v'_{EZ}[U_{T_N}])^2} \end{aligned}$$

and

$$\begin{aligned} \left(\frac{\partial \left(\frac{v'[\mu_t]}{v'[U_{T_N}]} \right)}{\partial R_{M,T_1,T_N}} \right)_{(x,y)=(x_0^*,y_0^*)} &= - \left\{ \frac{v_{EZ}[\mu_t] f''_v[v_{EZ}[\mu_t]]}{f'_v[v_{EZ}[\mu_t]]} \rho + \frac{v''_{EZ}[\mu_t] \mu_t}{v'_{EZ}[\mu_t]} \right\} \lambda_{1,t}^* \\ &= \left\{ \frac{1}{\tau_t^{f_v}} (1 - \gamma) + \gamma \right\} \lambda_{1,t}^*. \end{aligned} \quad (\text{II80})$$

Recall that

$$\frac{\partial \left(\frac{v'[\mu_t]}{v'[U_{T_N}]} \right)}{\partial R_{M,T_1,T_N}} = -f'_v[v_{EZ}[\mu_t]]v'_{EZ}[\mu_t] \frac{f''_v[v_{EZ}[U_{T_N}]]U'_{T_N}}{(f'_v[v_{EZ}[U_{T_N}]])^2} - f'_v[v_{EZ}[\mu_t]]v'_{EZ}[\mu_t] \frac{v''_{EZ}[U_{T_N}]U'_{T_N}}{f'_v[v_{EZ}[U_{T_N}]](v'_{EZ}[U_{T_N}])^2}$$

Hence

$$\begin{aligned}
& \frac{\partial^2 \left(\frac{v'[\mu_t]}{v'[U_{T_N}]} \right)}{\partial^2 R_{M,T_1,T_N}} \\
= & -f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{f'''_v[v_{EZ}[U_{T_N}]] v'_{EZ}[U_{T_N}] (U'_{T_N})^2}{(f'_v[v_{EZ}[U_{T_N}]])^2} - f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{f''_v[v_{EZ}[U_{T_N}]]}{(f'_v[v_{EZ}[U_{T_N}]])^2} \\
& + 2f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{(f''_v[v_{EZ}[U_{T_N}]])^2 v'_{EZ}[U_{T_N}] (U'_{T_N})^2}{(f'_v[v_{EZ}[U_{T_N}]])^3} \\
& - f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{v'''_{EZ}[U_{T_N}] (U'_{T_N})^2 + v''_{EZ}[U_{T_N}] U''_{T_N}}{f'_v[v_{EZ}[U_{T_N}]] (v'_{EZ}[U_{T_N}])^2} \\
& + f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{f''_v[v_{EZ}[U_{T_N}]] v'_{EZ}[U_{T_N}] v''_{EZ}[U_{T_N}] (U'_{T_N})^2}{(f'_v[v_{EZ}[U_{T_N}]])^2 (v'_{EZ}[U_{T_N}])^2} \\
& + 2f'_v[v_{EZ}[\mu_t]] v'_{EZ}[\mu_t] \frac{v'_{EZ}[U_{T_N}] v''_{EZ}[U_{T_N}] v'''_{EZ}[U_{T_N}] (U'_{T_N})^2}{f'_v[v_{EZ}[U_{T_N}]] (v'_{EZ}[U_{T_N}])^4}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{\partial^2 \left(\frac{v'[\mu_t]}{v'[U_{T_N}]} \right)}{\partial^2 R_{M,T_1,T_N}} \right)_{(x,y)=(x_0^*,y_0^*)} &= \left\{ \begin{aligned} & - (v'_{EZ}[\mu_t])^2 \frac{f'''_v[v_{EZ}[\mu_t]]}{f'_v[v_{EZ}[\mu_t]]} + 2 (v'_{EZ}[\mu_t])^2 \frac{(f''_v[v_{EZ}[\mu_t]])^2}{(f'_v[v_{EZ}[\mu_t]])^2} \\ & + \frac{f''_v[v_{EZ}[\mu_t]] v''_{EZ}[\mu_t]}{f'_v[v_{EZ}[\mu_t]]} + 2 \frac{v''_{EZ}[\mu_t] v'''_{EZ}[\mu_t]}{(v'_{EZ}[\mu_t])^2} - \frac{v'''_{EZ}[\mu_t]}{(v'_{EZ}[\mu_t])} \end{aligned} \right\} (\mu_t \lambda_{1,t}^* \\
& + \left\{ -v'_{EZ}[\mu_t] \frac{f''_v[v_{EZ}[\mu_t]]}{f'_v[v_{EZ}[\mu_t]]} - \frac{v''_{EZ}[\mu_t]}{(v'_{EZ}[\mu_t])} \right\} 2\mu_t \lambda_{2,t}
\end{aligned}$$

which simplifies to

$$\begin{aligned}
\left(\frac{\partial^2 \left(\frac{v'[\mu_t]}{v'[U_{TN}]} \right)}{\partial^2 R_{M,T_1,T_N}} \right)_{(x,y)=(x_0^*,y_0^*)} &= \left\{ \begin{aligned} &-(1-\gamma)^2 \frac{(v_{EZ}[\mu_t])^2 f_v'''[v_{EZ}[\mu_t]]}{f_v'[v_{EZ}[\mu_t]]} + 2(1-\gamma)^2 \frac{(v_{EZ}[\mu_t] f_v''[v_{EZ}[\mu_t]])^2}{(f_v'[v_{EZ}[\mu_t]])^2} \\ &+ \frac{f_v''[v_{EZ}[\mu_t]] v_{EZ}[\mu_t]}{f_v'[v_{EZ}[\mu_t]]} (-\gamma)(1-\gamma) + 2\gamma^2 - \gamma(\gamma+1) \end{aligned} \right\} \\
&+ \left\{ -(1-\gamma) \frac{v_{EZ}[\mu_t] f_v''[v_{EZ}[\mu_t]]}{f_v'[v_{EZ}[\mu_t]]} + \gamma \right\} 2\lambda_{2,t} \\
&= \left\{ 2 \frac{(1-\gamma)^2 (1-\rho_t^{f_v})}{(\tau_t^{f_v})^2} - \frac{1}{\tau_t^{f_v}} (-\gamma)(1-\gamma) + 2\gamma^2 - \gamma(\gamma+1) \right\} \lambda_1^* \\
&+ \left\{ (1-\gamma) \frac{1}{\tau_t^{f_v}} + \gamma \right\} 2\lambda_{2,t} \quad (\text{II81})
\end{aligned}$$

with

$$\rho_t^{f_v} = \frac{1}{2} \frac{f_v'''[v_{EZ}[\mu_t]] f_v'[v_{EZ}[\mu_t]]}{(f_v''[v_{EZ}[\mu_t]])^2}.$$

Finally, we combine (II75), (II78), (II80), and (II81), we simplify (II71) and obtain

$$\lambda = \frac{1}{2} (D_{1,t}^* + D_0^*) (\lambda_{1,t}^*)^2 + (D_{2,t}^* + D_{3,t}^*) \lambda_{2,t} \quad (\text{II82})$$

with

$$\begin{aligned}
D_0^* &= -\gamma \frac{1}{\psi} + \gamma(\gamma - 1) + \frac{1}{\psi} \left(1 + \frac{1}{\psi}\right) \\
D_{1,t}^* &= \frac{2(1 - \frac{1}{\psi})^2 \rho_t^{f_\vartheta}}{(\tau_t^{f_\vartheta})^2} + 2 \frac{(1 - \gamma)^2 (1 - \rho_t^{f_v})}{(\tau_t^{f_v})^2} - \frac{1}{\tau_t^{f_\vartheta}} \left(\frac{3}{\psi} + \gamma\right) \left(1 - \frac{1}{\psi}\right) \\
&\quad + \frac{1}{\tau_t^{f_v}} \left(\gamma(1 - \gamma) - \frac{1}{\psi}(1 - \gamma)\right) - \frac{1}{\tau_t^{f_v}} \frac{1}{\tau_t^{f_\vartheta}} (1 - \frac{1}{\psi})(1 - \gamma) \\
D_{2,t}^* &= (1 - \gamma) \frac{1}{\tau_t^{f_v}} - \left(1 - \frac{1}{\psi}\right) \frac{1}{\tau_t^{f_\vartheta}}, \\
D_{3,t}^* &= \gamma - \frac{1}{\psi}.
\end{aligned}$$

Using standard parameters,

$$\begin{aligned}
\gamma = 10, \psi = 2, D_{3,t}^* = 9.5 \text{ and } D_0^* &= \frac{343}{4} \\
\gamma = 10, \psi = 1, D_{3,t}^* = 9 \text{ and } D_0^* &= 82 \\
\gamma = 10, \psi = 3, D_{3,t}^* = \frac{29}{3} \text{ and } D_0^* &= \frac{784}{9}.
\end{aligned}$$

In the case of Epstein Zin specification, $\frac{1}{\tau_t^{f_\vartheta}} = \frac{1}{\tau_t^{f_v}} = 0$ hence $D_{1,t}^* = D_{2,t}^* = 0$. As $D_{3,t}^*$ and D_0^* are positive, $\lambda > 0$. However, when $D_{1,t}^*$ and $D_{2,t}^*$ are different from zero, one cannot conclude that $\lambda > 0$.

E. Bound for the equity risk premium (9)

Let us rewrite equation (I19) without truncation:

$$f[x, y] = 1 + \sum_{k=1}^{\infty} \sum_{m=0}^k \theta_{k,m} (x - x_0)^m (y - y_0)^{k-m} \quad (\text{II83})$$

where

$$\theta_{k,m} = \frac{1}{m!(k-m)!} \left. \frac{\partial^k f}{\partial x^m \partial y^{k-m}} \right|_{(x_0, y_0)}. \quad (\text{II84})$$

We have, in particular, $\theta_{1,1} = \frac{a_{1,t}}{R_{f,t,T_1}}$, $\theta_{1,0} = \frac{a_{1,t}}{R_{f,T_1,T_N}}$, $\theta_{2,2} = \frac{a_{2,t}}{R_{f,t,T_1}^2}$, $\theta_{2,0} = \frac{a_{2,t}}{R_{f,T_1,T_N}^2}$ and $\theta_{2,1} = \frac{a_{1,t} + 2a_{2,t}}{R_{f,t,T_N}}$.

The term that is truncated in the resulting second order expansion in equation (I24) is

$$\sum_{k=3}^{\infty} \sum_{m=0}^k \theta_{k,m} \underbrace{(R_{M,t,T_1} - R_{f,t,T_1})^m \mathbb{E}_{T_1}^* \left((R_{M,T_1,T_N} - R_{f,T_1,T_N})^{k-m} \right)}_{\text{Priced risk factor}}. \quad (\text{II85})$$

The $\theta_{k,m}$, $k \geq 3$, are functions of higher order preference coefficients. Thus, in our equity risk premium estimate, the truncated part of the numerator is

$$\sum_{k=3}^{\infty} \sum_{m=0}^k \theta_{k,m} \underbrace{\mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^{m+1} \mathbb{E}_{T_1}^* \left((R_{M,T_1,T_N} - R_{f,T_1,T_N})^{k-m} \right) \right)}_{\text{Risk premium associated to risk factor}} \quad (\text{II86})$$

and in the denominator it is

$$\sum_{k=3}^{\infty} \sum_{m=0}^k \theta_{k,m} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^m \mathbb{E}_{T_1}^* \left((R_{M,T_1,T_N} - R_{f,T_1,T_N})^{k-m} \right) \right). \quad (\text{II87})$$

Let us assume that Assumptions 1 and 2 hold. Under these assumptions, (II86) ≥ 0 and (II87) ≤ 0 , i.e., the second-order equity risk premium estimate in Equation (9) is in fact a lower bound to the equity risk premium.

F. Equity risk premium in a multi-period model with consumption

We now introduce consumption in the representative agent problem. We assume that assumptions 1 and 2 are satisfied. In addition, we assume that the consumption-wealth ratio is positively related to the market return, and that the correlation of the square of the consumption wealth ratio and market return is negative (condition reminiscent of market coskewness). We show that under these conditions, our measure of expected excess return remains a lower bound to the true expected excess return.

To proceed, we start by having the representative agent solve the problem

$$\max_{\omega_t, c_t} \mathbb{E}_t \left\{ \max_{\omega_{T_1}, c_{T_1}} \{ \mathbb{E}_{T_1} u [W_{t, T_N}] \} \right\}, \quad (\text{II88})$$

where the terminal wealth is

$$W_{t, T_N} = (1 - c_{T_1}) W_{T_1} (\omega_{T_1}^I R_{T_1, T_N}) \quad \text{with} \quad W_{T_1} = (1 - c_t) W_t (\omega_t^I R_{t, T_1}) \quad (\text{II89})$$

and c_t is the consumption wealth ratio. As in the case without consumption, the SDF is given by the identity (I18).

Let us define

$$cc_{tT_1} = (1 - c_{T_1}) (1 - c_t), \quad (\text{II90})$$

$$\mathbf{x} = cc_{tT_1}, \quad \mathbf{y} = R_{M, t, T_1}, \quad \mathbf{z} = R_{M, T_1, T_N} \quad (\text{II91})$$

$$\mathbf{x}_0 = 1, \quad \mathbf{y}_0 = R_{f, t, T_1}, \quad \mathbf{z}_0 = R_{f, T_1, T_N} \quad (\text{II92})$$

and $\mathbf{X} = (\mathbf{x}, \mathbf{y}, \mathbf{z})$ and $\mathbf{X}_0 = (\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$. Note that $0 < cc_{tT_1} \leq 1$ since $0 < c_{T_1} \leq 1$ and $0 < c_t \leq 1$. Now, assume that the utility function is well-behaved and admits high-order derivatives that exist. Denote $\bar{W}_{t,T_N} = W_t R_{f,t,T_1} R_{f,T_1,T_N}$ and

$$\mathbf{G} = \frac{u'[\bar{W}_{t,T_N}]}{u'[W_{t,T_N}]} \quad (\text{II93})$$

A second-order Taylor expansion of \mathbf{G} around $\mathbf{X} = \mathbf{X}_0$ gives

$$\begin{aligned} \mathbf{G} = & 1 - (\mathbf{x} - \mathbf{x}_0) \frac{W_t \mathbf{y}_0 \mathbf{z}_0 u''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} - (\mathbf{y} - \mathbf{y}_0) \frac{W_t \mathbf{z}_0 u''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} - (\mathbf{z} - \mathbf{z}_0) \frac{W_t \mathbf{y}_0 u''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} \\ & + \frac{1}{2} W_t^2 \mathbf{y}_0^2 \mathbf{z}_0^2 \left(-\frac{u''''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} + \frac{2(u''[\bar{W}_{t,T_N}])^2}{(u'[\bar{W}_{t,T_N}])^2} \right) (\mathbf{x} - \mathbf{x}_0)^2 \\ & + \frac{1}{2} W_t^2 \mathbf{z}_0^2 \left(-\frac{u''''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} + \frac{2(u''[\bar{W}_{t,T_N}])^2}{(u'[\bar{W}_{t,T_N}])^2} \right) (\mathbf{y} - \mathbf{y}_0)^2 \\ & + \frac{1}{2} W_t^2 \mathbf{y}_0^2 \left(-\frac{u''''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} + \frac{2(u''[\bar{W}_{t,T_N}])^2}{(u'[\bar{W}_{t,T_N}])^2} \right) (\mathbf{z} - \mathbf{z}_0)^2 \\ & + W_t^2 \mathbf{y}_0 \mathbf{x}_0 \mathbf{z}_0^2 \left(-\frac{u''''[\bar{W}_{t,T_N}]}{u'[\bar{W}_{t,T_N}]} + \frac{2(u''[\bar{W}_{t,T_N}])^2}{(u'[\bar{W}_{t,T_N}])^2} \right) (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) \\ & + \left(\frac{\partial^2 \mathbf{G}}{\partial \mathbf{x} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{x} - \mathbf{x}_0) (\mathbf{z} - \mathbf{z}_0) + \left(\frac{\partial^2 \mathbf{G}}{\partial \mathbf{y} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{z} - \mathbf{z}_0) (\mathbf{y} - \mathbf{y}_0). \end{aligned} \quad (\text{II94})$$

Notice that

$$\mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0) = 0 \quad (\text{II95})$$

and

$$\begin{aligned} \mathbb{E}_{T_1}^* (\mathbf{x} - \mathbf{x}_0) (\mathbf{z} - \mathbf{z}_0) &= (\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0) = 0, \\ \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0) (\mathbf{y} - \mathbf{y}_0) &= (\mathbf{y} - \mathbf{y}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0) = 0. \end{aligned} \quad (\text{II96})$$

We use these expressions to compute v_{T_1} , defined as in (3):

$$\begin{aligned}
v_{T_1} &= \mathbb{E}_{T_1}^* \left(\frac{u' [\overline{W}_{t,T_N}]}{u' [W_{t,T_N}]} \right) = 1 - \frac{W_t \mathbf{y}_0 \mathbf{z}_0 u'' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} (\mathbf{x} - \mathbf{x}_0) - (\mathbf{y} - \mathbf{y}_0) \frac{W_t \mathbf{z}_0 u'' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} \\
&+ \frac{1}{2} W_t^2 \mathbf{y}_0^2 \mathbf{z}_0^2 \left(-\frac{u''' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} + \frac{2 (u'' [\overline{W}_{t,T_N}])^2}{(u' [\overline{W}_{t,T_N}])^2} \right) (\mathbf{x} - \mathbf{x}_0)^2 \\
&+ \frac{1}{2} W_t^2 \mathbf{z}_0^2 \left(-\frac{u''' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} + \frac{2 (u'' [\overline{W}_{t,T_N}])^2}{(u' [\overline{W}_{t,T_N}])^2} \right) (\mathbf{y} - \mathbf{y}_0)^2 \\
&+ \frac{1}{2} W_t^2 \mathbf{y}_0^2 \left(-\frac{u''' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} + \frac{2 (u'' [\overline{W}_{t,T_N}])^2}{(u' [\overline{W}_{t,T_N}])^2} \right) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\
&+ W_t^2 \mathbf{y}_0 \mathbf{x}_0 \mathbf{z}_0^2 \left(-\frac{u''' [\overline{W}_{t,T_N}]}{u' [\overline{W}_{t,T_N}]} + \frac{2 (u'' [\overline{W}_{t,T_N}])^2}{(u' [\overline{W}_{t,T_N}])^2} \right) (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0)
\end{aligned} \tag{II97}$$

which simplifies to

$$\begin{aligned}
v_{T_1} &= 1 + \frac{1}{\tau_t} \mathbb{E}_{T_1}^* (cc_{tT_1} - 1) + \frac{1}{\tau_t R_{f,t,T_1}} (\omega_t^\top R_{t,T_1} - R_{f,t,T_1}) + \frac{(1 - \rho_t)}{\tau_t^2} (cc_{tT_1} - 1)^2 \\
&+ \frac{(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}^2} (\omega_t^\top R_{t,T_1} - R_{f,t,T_1})^2 + \frac{(1 - \rho_t)}{\tau_t^2 R_{f,T_1,T_N}^2} \mathbb{E}_{T_1}^* (\omega_{T_1}^\top R_{T_1,T_N} - R_{f,T_1,T_N})^2 \\
&+ \frac{2(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}} \mathbb{E}_{T_1}^* (cc_{tT_1} - 1) (\omega_t^\top R_{t,T_1} - R_{f,t,T_1}).
\end{aligned} \tag{II98}$$

We then use $R_{M,t,T_1} = \omega_t^\top R_{t,T_1}$, $R_{M,T_1,T_N} = \omega_{T_1}^\top R_{T_1,T_N}$ and express the expected value of v_{T_1} under the risk neutral measure as

$$\begin{aligned}
\mathbb{E}_t^* v_{T_1} &= 1 + \frac{1}{\tau_t} \mathbb{E}_t^* (cc_{tT_1} - 1) + \frac{(1 - \rho_t)}{\tau_t^2} \mathbb{E}_t^* (cc_{tT_1} - 1)^2 \\
&+ \frac{(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1 - \rho_t)}{\tau_t^2 R_{f,T_1,T_N}^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \\
&+ \frac{2(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}} \text{COV}_t^* (cc_{tT_1}, R_{M,t,T_1}).
\end{aligned} \tag{II99}$$

The expected excess market return is

$$\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1}) = \mathbb{E}_t \left[\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \frac{m_{t,T_1}}{\mathbb{E}_t m_{t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1}) \right] \quad (\text{II100})$$

$$\begin{aligned} &= \mathbb{E}_t^* \left[\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1}) \right] \\ &= \frac{\text{COV}_t^* [v_{T_1}, R_{M,t,T_1}]}{\mathbb{E}_t^* v_{T_1}}. \end{aligned} \quad (\text{II101})$$

Observe that

$$\begin{aligned} \text{COV}_t^* [v_{T_1}, R_{M,t,T_1}] &= 1 + \frac{1}{\tau_t} \text{COV}_t^* (cc_{tT_1}, R_{M,t,T_1}) + \frac{1}{\tau_t R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(2)} \\ &\quad + \frac{(1 - \rho_t)}{\tau_t^2} \text{COV}_t^* ((cc_{tT_1} - 1)^2, R_{M,t,T_1}) + \frac{(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(3)} \\ &\quad + \frac{(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}^2} \mathbb{L} \mathbb{E} v_{t,T_1,T_N}^* \\ &\quad + \frac{2(1 - \rho_t)}{\tau_t^2 R_{f,t,T_1}^2} \mathbb{E}_t^* ((cc_{tT_1} - 1) (R_{M,t,T_1} - R_{f,t,T_1})^2). \end{aligned} \quad (\text{II102})$$

Notice that $\mathbb{E}_t^* \left((cc_{tT_1} - 1) (\omega_t^I R_{t,T_1} - R_{f,t,T_1})^2 \right) < 0$ because $cc_{tT_1} - 1 < 0$.

In theory, each factor risk premium in v_{T_1} positively contributes to the equity risk premium, therefore each term in (II102) is positive. Thus one should expect

$$\text{COV}_t^* (cc_{tT_1}, R_{M,t,T_1}) > 0 \text{ and } \text{COV}_t^* ((cc_{tT_1} - 1)^2, R_{M,t,T_1}) \leq 0. \quad (\text{II103})$$

Since $1 - c_{T_1} = \frac{W_{T_1} - C_{T_1}}{W_{T_1}}$ is the fraction of wealth W_{T_1} invested at T_1 , it follows

that

$$\begin{aligned} \mathbb{C}\text{OV}_t^*(cc_{tT_1}, R_{M,t,T_1}) &= (1 - c_t) \mathbb{C}\text{OV}_t^*\left(\frac{W_{T_1} - C_{T_1}}{W_{T_1}}, R_{M,t,T_1}\right) \text{ and} \\ \mathbb{C}\text{OV}_t^*((cc_{tT_1} - 1)^2, R_{M,t,T_1}) &= (1 - c_t)^2 \mathbb{C}\text{OV}_t^*\left(\left(\frac{W_{T_1} - C_{T_1}}{W_{T_1}}\right)^2, R_{M,t,T_1}\right). \end{aligned} \quad (\text{II104})$$

The positive sign of $\mathbb{C}\text{OV}_t^*(cc_{tT_1}, R_{M,t,T_1})$ is motivated by the positive impact of wealth-consumption ratio on the market expected excess return. Conditions (II103) are reminiscent of the dependence between the wealth-consumption ratio and the return on the market under the physical measure. Under the physical measure, the wealth-consumption ratio is positively correlated to the market. Under Assumptions 1 and 2, and (II103), the covariance $\mathbb{C}\text{OV}_t^*[v_{T_1}, R_{M,t,T_1}]$ is bounded:

$$\mathbb{C}\text{OV}_t^*[v_{T_1}, R_{M,t,T_1}] \geq \frac{1}{R_{f,t,T_1}} \frac{1}{\tau_t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1 - \rho_t)}{R_{f,t,T_1}^2 \tau_t^2} \mathbb{M}_{t,T_1}^{*(3)} + \frac{(1 - \rho_t)}{R_{f,T_1,T_N}^2 \tau_t^2} \mathbb{L}\mathbb{E}\mathbb{V}_{t,T_1,T_N}^*. \quad (\text{II105})$$

Next, since $cc_{tT_1} \leq 1$, we use (II99) and (II103) to obtain

$$\mathbb{E}_t^* v_{T_1} \leq 1 + \frac{(1 - \rho_t)}{R_{f,t,T_1}^2 \tau_t^2} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1 - \rho_t)}{R_{f,T_1,T_N}^2 \tau_t^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)}. \quad (\text{II106})$$

Therefore,

$$\frac{1}{\mathbb{E}_t^* v_{T_1}} \geq \frac{1}{1 + \frac{(1 - \rho_t)}{R_{f,t,T_1}^2 \tau_t^2} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1 - \rho_t)}{R_{f,T_1,T_N}^2 \tau_t^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)}}. \quad (\text{II107})$$

Combining (II105) and (II107), the expected excess return is bounded by our

equity risk premium estimate:

$$\mathbb{E}_t [R_{M,t,T_1} - R_{f,t,T_1}] \geq \frac{\frac{1}{R_{f,t,T_1}} \frac{1}{\tau_t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1-\rho_t)}{R_{f,t,T_1}^2 \tau_t^2} \mathbb{M}_{t,T_1}^{*(3)} + \frac{(1-\rho_t)}{R_{f,T_1,T_N}^2 \tau_t^2} \mathbb{LEV}_{t,T_1,T_N}^*}{1 + \frac{(1-\rho_t)}{R_{f,t,T_1}^2 \tau_t^2} \mathbb{M}_{t,T_1}^{*(2)} + \frac{(1-\rho_t)}{R_{f,T_1,T_N}^2 \tau_t^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)}} \quad (\text{II108})$$

This shows that under minimal conditions, our measure of expected excess return is a bound on the true expected excess return when consumption is taken into account.

G. Restricted bound (23)

Let us assume that Assumptions 3–4 hold. Under these conditions, $a_{1,t} \geq 1$ and $a_{2,t} \leq -1$. Hence,

$$\frac{a_{1,t}}{R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(2)} \geq \frac{1}{R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(2)} \quad (\text{II109})$$

$$\frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(3)} \geq \frac{1}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(3)} \quad (\text{II110})$$

$$\frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{LEV}_{t,T_1,T_N}^* \geq \frac{1}{R_{f,T_1,T_N}^2} \mathbb{LEV}_{t,T_1,T_N}^* \quad (\text{II111})$$

which shows that the numerator of (9) is larger than the numerator of (23).

Furthermore,

$$\frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(2)} \leq \frac{-1}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(2)} \quad (\text{II112})$$

$$\frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \leq \frac{-1}{R_{f,T_1,T_N}^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \quad (\text{II113})$$

which shows that the denominator of (9) is smaller than the denominator of (23).

The inequality follows.

III . Estimation of moments

In this Section, we provide closed-form solutions to the risk-neutral and physical moments used in our analysis. In many cases, we use the spanning formula of Carr and Madan (2001) and Bakshi and Madan (2000) to evaluate the risk-neutral expected value of a twice-differentiable function of the underlying asset price, $H(S_{T_1})$ as

$$\begin{aligned} \mathbb{E}_t^* H[S_{T_1}] &= H[S_t R_{f,t,T_1}] + \mathbb{E}_t^* H_S[S_t R_{f,t,T_1}] S_t (R_{M,t,T_1} - R_{f,t,T_1}) \\ &\quad + R_{f,t,T_1} \left[\int_{S_t R_{f,t,T_1}}^{\infty} H_{SS}[K] C_t[K] dK + \int_0^{S_t R_{f,t,T_1}} H_{SS}[K] P_t[K] dK \right], \end{aligned} \tag{III1}$$

where H_S and H_{SS} are the first and second derivative of function $H(\cdot)$, respectively, and $C_t[K]$ and $P_t[K]$ are the prices of call and put options with strike K and maturity T_1 at time t . We evaluate the integral terms via numerical integration using the 1,000-point moneyness grid described in Section IV.B.

A. Closed-form expressions for $\mathbb{M}_{t,T_1}^{*(k)}$ and $\mathbb{M}_{t,T_1}^{*(k)}[\alpha]$

Let us set $H(x) = \left(\frac{x}{S_t} - R_{f,t,T_1}\right)^k$. To evaluate the risk-neutral moments of order k , $\mathbb{M}_{t,T_1}^{*(k)}$, we plug H in Equation (III1). The risk-neutral moments are therefore computed using call and put options with maturity T_1 .

To compute the truncated moments $\mathbb{M}_{t,T_1}^{*(k)}[\alpha] = \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^k \mathbb{1}_{S_{T_1} < \alpha S_t} \right)$, we use $G(x) = H(x) \mathbb{1}_{x < \alpha S_t}$. Equation (III1) simplifies to

$$\mathbb{M}_{t,T_1}^{*(k)}[\alpha] = H[\alpha S_t] \mathbb{P}_t^*[S_{T_1} < \alpha S_t] - H_S[\alpha S_t] R_{f,t,T_1} P_t[\alpha S_t] + R_{f,t,T_1} \int_0^{\alpha S_t} H_{SS}[K] P_t[K] dK. \quad (\text{III2})$$

B. Representation of $\mathbb{M}_{T_1,T_N}^{*(2)}$ as a function of S_{T_1}

Let us decompose the variance of market returns between T_1 and T_N into a portfolio of options, using the spanning formula of Carr and Madan (2001) and Bakshi and Madan (2000) with $H(x) = \left(\frac{x}{S_{T_1}} - R_{f,T_1,T_N} \right)^2$:

$$\mathbb{M}_{T_1,T_N}^{*(2)} = \mathbb{E}_{T_1}^* \left(\left(\frac{S_{T_N}}{S_{T_1}} - R_{f,T_1,T_N} \right)^2 \right) \quad (\text{III3})$$

$$= \frac{2R_{f,T_1,T_N}}{S_{T_1}^2} \left[\int_{R_{f,T_1,T_N} S_{T_1}}^{\infty} C_{T_1}[K] dK + \int_0^{R_{f,T_1,T_N} S_{T_1}} P_{T_1}[K] dK \right] \quad (\text{III4})$$

where $C_{T_1}[K]$ and $P_{T_1}[K]$ denote the prices of call and put options with strike K and maturity T_N at time T_1 . For each value of $K = K_0$, $C_{T_1}[K]$ and $P_{T_1}[K]$ can be expressed as functions of S_{T_1} , R_{f,T_1,T_N} , $T_N - T_1$ and $\sigma_{IV}^2(k_0)$, where $\sigma_{IV}^2(k_0)$ is the implied variance of the option at time T_1 . The implied variance is evaluated at the log-moneyness $k_0 = \ln(K_0/F_{T_1,T_N})$, where F_{T_1,T_N} is the forward price.

Consider, for example, the arbitrage-free stochastic volatility inspired (raw) parameterization of the volatility surface proposed by Gatheral and Jacquier (2014). It writes the total implied variance evaluated at the log-

moneyness k_0 as a function of a set of parameters $\{a, b, \xi, m, \sigma\}$:

$$\sigma_{IV}^2(k_0)(T_N - T_1) = a + b \left(\xi(k_0 - m) + \sqrt{(k_0 - m)^2 + \sigma^2} \right) \quad (\text{III5})$$

where $a \in \mathbb{R}$, $b \geq 0$, $|\xi| < 1$, $m \in \mathbb{R}$, $\sigma > 0$, and $a + b\sigma\sqrt{1 - \xi^2} \geq 0$. Given these parameters, the option prices $C_{T_1}[K]$ and $P_{T_1}[K]$ are thus functions of S_{T_1} .

C. Volatility dynamics implied by (32)

To show that our formulation (32) is different from the GARCH (1,1) model, we use the closed-form expression of θ_{T_1, T_N} displayed in (35) to obtain

$$\mathbb{M}_{t, T_N}^{*(2)} = \theta_{T_1, T_N} \mathbb{M}_{t, T_1}^{*(4)} + 2R_{f, t, T_1} \theta_{T_1, T_N} \mathbb{M}_{t, T_1}^{*(3)} + R_{f, T_1, T_N}^2 (\theta_{T_1, T_N} + 1) \mathbb{M}_{t, T_1}^{*(2)}. \quad (\text{III6})$$

This proves that $\mathbb{M}_{t, T_N}^{*(2)}$ differs from the GARCH dynamics. To check similarities with the GARCH process, let us assume for illustration purposes that $\mathbb{M}_{t, T_1}^{*(3)} = 0$ and $\mathbb{M}_{t, T_1}^{*(4)} = 3 \left(\mathbb{M}_{t, T_1}^{*(2)} \right)^2$. Then

$$\mathbb{M}_{t, T_N}^{*(2)} = 3\theta_{T_1, T_N} \left(\mathbb{M}_{t, T_1}^{*(2)} \right)^2 + R_{f, T_1, T_N}^2 (\theta_{T_1, T_N} + 1) \mathbb{M}_{t, T_1}^{*(2)}. \quad (\text{III7})$$

The propagation of volatility is therefore quadratic in the current variance, unlike the affine GARCH recursion in which it is linear.

D. Closed-form expressions for $\mathbb{E}_t^ \mathbb{M}_{T_{j-1}, T_j}^{*(2)}$ and $\mathbb{L}\mathbb{E}\mathbb{V}_{t, T_{j-1}, T_j}^*$*

To compute the risk neutral quantities, we use an approach similar to (32) by considering the decomposition:

$$\mathbb{M}_{T_{j-1}, T_j}^{*(2)} = \theta_{T_{j-1}, T_j} (R_{M, t, T_{j-1}} - R_{f, t, T_{j-1}})^2 + \eta_{T_{j-1}} \quad (\text{III8})$$

with $\mathbb{E}^* (\eta_{T_{j-1}} | R_{M, t, T_{j-1}}) = 0$. The loadings θ_{T_{j-1}, T_j} can be computed at time t from options with maturities T_{j-1} and T_j :

$$\theta_{T_{j-1}, T_j} = \frac{\mathbb{M}_{t, T_j}^{*(2)} - R_{f, T_{j-1}, T_j}^2 \mathbb{M}_{t, T_{j-1}}^{*(2)}}{\mathbb{E}_t^* \left(R_{M, t, T_{j-1}}^2 (R_{M, t, T_{j-1}} - R_{f, t, T_{j-1}})^2 \right)}. \quad (\text{III9})$$

Our objective is to calculate the moments $\mathbb{E}_t^* \left((R_{M, t, T_1} - R_{f, t, T_1})^k \mathbb{M}_{T_{j-1}, T_j}^{*(2)} \right)$. For $k = 0$, we obtain $\mathbb{E}_t^* (\mathbb{M}_{T_{j-1}, T_j}^{*(2)})$ which appears in the denominator of the equity risk premium and of the physical variance. For $k = 1$, we obtain the additional term in the numerator of the equity risk premium, which arises due to intertemporal hedging. For $k = 2$, we obtain the additional term in the physical variance.

If $T_{j-1} = T_1$, using (III8) we get

$$\mathbb{E}_t^* \left((R_{M, t, T_1} - R_{f, t, T_1})^k \mathbb{M}_{T_1, T_j}^{*(2)} \right) = \theta_{T_1, T_j} \mathbb{M}_{t, T_1}^{*(k+2)}. \quad (\text{III10})$$

For $T_{j-1} > 1$, using (III8) and

$$\mathbb{M}_{T_1, T_{j-1}}^{*(2)} = \theta_{T_1, T_{j-1}} (R_{M, t, T_1} - R_{f, t, T_1})^2 + \eta_{T_1} \text{ with } \mathbb{E}_t^* (\eta_{T_1} | R_{M, t, T_1}) = 0, \quad (\text{III11})$$

we obtain

$$\begin{aligned} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^k \mathbb{M}_{T_{j-1},T_j}^{*(2)} \right) &= \theta_{T_1,T_{j-1}} \theta_{T_{j-1},T_j} \left(\mathbb{M}_{t,T_1}^{*(k+4)} + 2R_{f,t,T_1} \mathbb{M}_{t,T_1}^{*(k+3)} + R_{f,t,T_1}^2 \mathbb{M}_{t,T_1}^{*(k+2)} \right) \\ &\quad + \theta_{T_{j-1},T_j} R_{f,T_1,T_{j-1}}^2 \mathbb{M}_{t,T_1}^{*(k+2)}. \end{aligned} \quad (\text{III12})$$

For $k = 0$, (III12) collapses to

$$\mathbb{E}_t^* \mathbb{M}_{T_{j-1},T_j}^{*(2)} = \theta_{T_{j-1},T_j} \mathbb{M}_{t,T_{j-1}}^{*(2)}. \quad (\text{III13})$$

For $k = 1$,

$$\begin{aligned} \mathbb{L}\mathbb{E}\mathbb{V}_{t,T_{j-1},T_j}^* &= \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{M}_{T_{j-1},T_j}^{*(2)} \right) \\ &= \theta_{T_{j-1},T_j} \left(\theta_{T_1,T_{j-1}} \mathbb{M}_{t,T_1}^{*(5)} + 2\theta_{T_1,T_{j-1}} R_{f,t,T_1} \mathbb{M}_{t,T_1}^{*(4)} + (\theta_{T_1,T_{j-1}} R_{f,t,T_1}^2 + R_{f,T_1,T_{j-1}}^2) \mathbb{M}_{t,T_1}^{*(3)} \right). \end{aligned} \quad (\text{III14})$$

For $k = 2$,

$$\begin{aligned} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_{j-1},T_j}^{*(2)} \right) &= \theta_{T_1,T_{j-1}} \theta_{T_{j-1},T_j} \left(\mathbb{M}_{t,T_1}^{*(6)} + 2R_{f,t,T_1} \mathbb{M}_{t,T_1}^{*(5)} + R_{f,t,T_1}^2 \mathbb{M}_{t,T_1}^{*(4)} \right) \\ &\quad + \theta_{T_{j-1},T_j} R_{f,T_1,T_{j-1}}^2 \mathbb{M}_{t,T_1}^{*(4)}. \end{aligned} \quad (\text{III15})$$

E. Closed-form expression of $\mathbb{E}_t^ \left(\mathbb{M}_{T_{j-1},T_j}^{*(2)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right)$*

If $T_{j-1} = T_1$, using (III8) we obtain

$$\mathbb{E}_t^* \left[\mathbb{M}_{T_{j-1},T_j}^{*(2)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right] = \theta_{T_1,T_j} \mathbb{M}_{t,T_1}^{*(2)} [\alpha] \quad (\text{III16})$$

In the model with single rebalancing time, $T_j = T_N$ and the intertemporal hedging component of the probability of a crash is given by

$$\mathbb{E}_t^* \left(\mathbb{M}_{T_1, T_N}^{*(2)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right) = \theta_{T_1, T_N} \mathbb{M}_{t, T_1}^{*(2)} [\alpha]. \quad (\text{III17})$$

If $T_{j-1} > T_1$, we obtain

$$\begin{aligned} \mathbb{E}_t^* \left[\mathbb{M}_{T_{j-1}, T_j}^{*(2)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right] &= \theta_{T_{j-1}, T_j} \mathbb{E}_t^* \left[\left(R_{M,t,T_{j-1}} - R_{f,t,T_{j-1}} \right)^2 \mathbb{1}_{R_{M,t,T_1} < \alpha} \right] \\ &= \theta_{T_{j-1}, T_j} \left\{ \begin{array}{l} \mathbb{E}_t^* \left(R_{M,t,T_1}^2 \mathbb{1}_{R_{M,t,T_1} < \alpha} \left(\mathbb{M}_{T_1, T_{j-1}}^{*(2)} + R_{f,T_1, T_{j-1}}^2 \right) \right) \\ - 2R_{f,t,T_{j-1}} R_{f,T_1, T_{j-1}} \left(\mathbb{M}_{t, T_1}^{*(1)} [\alpha] + R_{f,t,T_1} \mathbb{M}_{t, T_1}^{*(0)} [\alpha] \right) \\ + R_{f,t,T_{j-1}}^2 \mathbb{M}_{t, T_1}^{*(0)} [\alpha] \end{array} \right\} \\ &= \theta_{T_{j-1}, T_j} \left\{ \begin{array}{l} \theta_{T_1, T_{j-1}} \left(\mathbb{M}_{t, T_1}^{*(4)} [\alpha] + 2R_{f,t,T_1} \mathbb{M}_{t, T_1}^{*(3)} [\alpha] + R_{f,t,T_1}^2 \mathbb{M}_{t, T_1}^{*(2)} [\alpha] \right) \\ + R_{f,T_1, T_{j-1}}^2 \left(\mathbb{M}_{t, T_1}^{*(2)} [\alpha] + 2R_{f,t,T_1} \mathbb{M}_{t, T_1}^{*(1)} [\alpha] + R_{f,t,T_1}^2 \mathbb{M}_{t, T_1}^{*(0)} [\alpha] \right) \\ - 2R_{f,t,T_{j-1}} R_{f,T_1, T_{j-1}} \left(\mathbb{M}_{t, T_1}^{*(1)} [\alpha] + R_{f,t,T_1} \mathbb{M}_{t, T_1}^{*(0)} [\alpha] \right) \\ + R_{f,t,T_{j-1}}^2 \mathbb{M}_{t, T_1}^{*(0)} [\alpha] \end{array} \right\} \end{aligned} \quad (\text{III18})$$

IV . Out-of-sample performance over the sample used in Tetlock, McCoy, and Shah (2024)

In this section, we reproduce the out-of-sample performance results but on the sample from 1997 to 2021. Table IAI presents the summary statistics of the data over this time period and Table IAII reports the out-of-sample

prediction performance.

Table IAI
Summary statistics for risk premia from 1997 to 2021

We report summary statistics for times series of risk premia predictions. We use weekly time series of overlapping values for horizons longer than 10 days. Each panel reports the results for a forecast horizon T_1 between one week and six months. All values are annualized and in percent. RP_{t,T_1}^{Log} is the lower bound of Martin (2017). $IERP_{t,T_1}$ is the risk premium estimate of Tetlock, McCoy, and Shah (2024). RP_{t,T_1} is the second-order lower bound of Chabi-Yo and Loudis (2020). RP_{t,T_1,T_N} is the risk premium measure in Equation (12), with $T_N = 1$ and 2 years. Data are weekly from January 1997 to December 2021.

Prediction	Mean	Standard deviation	Skew.	Kurt.	10%	25%	50%	75%	90%
<i>Panel A: One week</i>									
RP_{t,T_1}^{Log}	3.43	4.87	6.29	59.08	1.01	1.36	2.05	3.55	6.19
$IERP_{t,T_1}$	8.37	13.40	7.20	75.05	2.35	3.12	4.75	8.38	14.55
RP_{t,T_1}	3.59	5.30	6.70	66.84	1.04	1.41	2.12	3.66	6.49
$RP_{t,T_1,1y}$	9.33	10.02	6.96	82.58	3.70	4.58	6.59	10.56	15.88
$RP_{t,T_1,2y}$	14.75	11.45	5.57	60.09	6.83	8.71	11.53	17.12	24.11
<i>Panel B: One month</i>									
RP_{t,T_1}^{Log}	4.29	4.38	4.57	34.44	1.32	1.82	3.21	5.10	7.85
$IERP_{t,T_1}$	8.74	11.11	5.69	47.26	2.46	3.66	5.88	9.82	15.32
RP_{t,T_1}	4.60	4.89	4.84	38.09	1.37	1.92	3.43	5.40	8.48
$RP_{t,T_1,1y}$	7.69	6.87	4.33	33.92	2.72	3.83	5.93	9.20	13.45
$RP_{t,T_1,2y}$	10.11	7.56	3.63	26.09	4.18	5.59	8.15	12.20	17.05
<i>Panel C: One quarter</i>									
RP_{t,T_1}^{Log}	4.35	3.39	3.39	21.22	1.68	2.15	3.51	5.32	7.52
$IERP_{t,T_1}$	9.11	8.81	4.45	31.42	3.26	4.56	6.80	10.51	15.85
RP_{t,T_1}	4.88	3.98	3.71	25.32	1.85	2.41	3.86	5.91	8.52
$RP_{t,T_1,1y}$	6.42	4.75	3.54	24.66	2.56	3.48	5.23	7.68	10.93
$RP_{t,T_1,2y}$	7.64	5.03	3.35	22.94	3.60	4.62	6.49	9.08	12.69
<i>Panel D: Six months</i>									
RP_{t,T_1}^{Log}	4.32	2.68	2.61	14.02	1.94	2.45	3.70	5.33	7.02
$IERP_{t,T_1}$	9.22	7.06	3.47	20.12	3.90	5.21	7.32	10.81	15.76
RP_{t,T_1}	5.02	3.23	2.81	15.99	2.20	2.84	4.28	6.10	8.31
$RP_{t,T_1,1y}$	6.18	3.78	2.57	13.96	2.82	3.66	5.29	7.38	10.35
$RP_{t,T_1,2y}$	7.05	4.02	2.48	13.35	3.50	4.46	6.11	8.42	11.49

Table I AII
Out-of-sample prediction and allocation performance from 1997 to 2021

We report the out-of-sample performance of different risk premium prediction methods. RP_{t,T_1}^{Log} is the lower bound of Martin (2017). $IERP_{t,T_1}$ is the Implied Equity Risk Premium of Tetlock, McCoy, and Shah (2024). RP_{t,T_1} is the second-order lower bound of Chabi-Yo and Loutis (2020). RP_{t,T_1,T_N} is the risk premia measure in Equation (12). We report in Panel A the out-of-sample prediction R_{OOS}^2 in percent (see Equation (39)). The results in the last column are based on predicted returns obtained by averaging RP_{t,T_1,T_N} across T_N . For each prediction method, we test for the significance of the R_{OOS}^2 difference relative to RP_{t,T_1} using a Diebold and Mariano (1995) test. We estimate the variance of the differences using a Newey-West correction with 12 lags. We report in Panel B the realized mean-variance certainty equivalents using each period the predicted risk premium and physical variance to obtain the optimal allocation (see Equation (41)). The physical variances are computed using option prices, using Equation (28). For each prediction method, we test for the significance of the realized certainty equivalent difference relative to RP_{t,T_1} using a block-bootstrap with average block length of three years and 10,000 bootstraps. Realized certainty equivalents are computed from non-overlapping returns. Negative certainty equivalents are not reported. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively. Data are from January 1997 to December 2021.

Horizon T_1		RP_{t,T_1,T_N} with $T_N =$ (in months)												Average	
(in months)	RP_{t,T_1}^{Log}	$IERP_{t,T_1}$	RP_{t,T_1}	1	2	3	4	5	6	9	12	18	24	across T_N	
<i>Panel A: Out-of-sample R^2</i>															
10d	-0.41	-0.60	-0.38	-0.33	-0.21	-0.11	-0.00	0.10	0.21	0.46	0.65	0.91	0.95	0.43	
1	0.92	1.39	1.08	-	1.25	1.52	1.61	1.70	1.79	1.97	2.11	2.30	2.34	1.95	
2	1.51	2.18	1.96	-	-	2.23*	2.70*	2.87*	3.03*	3.42**	3.72**	4.20**	4.48*	3.43**	
3	1.43	2.73	2.23	-	-	-	2.55**	3.14**	3.37**	3.90**	4.31**	4.95**	5.38**	4.04**	
4	2.17	5.22	3.35	-	-	-	-	3.69**	4.31**	4.96**	5.45**	6.19**	6.69**	5.31**	
5	3.07	8.01	4.65	-	-	-	-	-	5.00***	6.19***	6.73**	7.54**	8.09**	6.80***	
6	3.40	9.42	5.28	-	-	-	-	-	-	6.23***	7.74***	8.59***	9.18***	8.02***	
12	2.67	10.38	5.58	-	-	-	-	-	-	-	-	6.90***	9.03***	8.03***	
<i>Panel B: Out-of-sample mean-variance certainty equivalent with $\gamma = 3$</i>															
10d	3.66	4.59	3.74	3.92*	4.42*	4.77*	5.17*	5.56*	5.95*	6.73*	7.28	7.30	7.88	6.80*	
1	4.71	2.91	4.89	-	5.11*	5.58	5.80	6.03	6.28*	6.94	7.49	8.29	9.17	7.03*	
2	4.99	3.54	5.39	-	-	5.67**	6.27**	6.53**	6.79**	7.53***	8.28**	9.75**	10.27**	7.67***	
3	5.53	7.30	6.13	-	-	-	6.41**	6.99**	7.24**	7.99**	8.77**	10.48**	11.82**	8.41**	
4	5.71	5.93	6.44	-	-	-	-	6.70***	7.25***	7.96**	8.66**	10.17**	11.39**	8.56**	
5	5.42	4.31	6.16	-	-	-	-	-	6.35**	7.20**	7.75**	8.93**	10.05**	7.98**	
6	5.58	2.91	6.59	-	-	-	-	-	-	7.31***	9.12***	10.69***	11.50**	9.77***	
12	5.40	-	6.42	-	-	-	-	-	-	-	-	7.21	7.62	7.88	

V . Validity and tightness tests

We follow the methodology of Back, Crotty, and Kazempour (2022) and test for the validity and tightness of these bounds. We recall below the methodology.

The test is based on the set of inequalities

$$\mathbb{E}_t[(R_{M,t,T_1} - R_{f,t,T_1}) - \tilde{r}_{M,t,T_1}]z_t \geq 0$$

for a vector z_t of nonnegative conditioning variables in the time- t information set. Back, Crotty, and Kazempour (2022) refer to $((R_{M,t,T_1} - R_{f,t,T_1}) - \tilde{r}_{M,t,T_1})$ as realized slackness. We consider $\tilde{r}_{M,t,T_1} \in \{RP_{t,T_1}^{Log}, RP_{t,T_1}, RP_{t,T_1,T_N^*}\}$. The vector z_t contains variables from Welch and Goyal (2008): Dividend Price Ratio (defined as the difference between the log of dividends and the log of the S&P 500 index price level, plus 5 to ensure a positive conditioning variable); Earnings Price Ratio (defined as the difference between the log of earnings and the log of the S&P 500 index price level, plus 5 to ensure a positive conditioning variable); Book-to-Market Ratio (the ratio of book value to market value for the Dow Jones Industrial Average); T-bill Rate (the 3-month Treasury bill rate); 1 + Term Spread (defined as the difference between the long-term yield from Ibbotson's and the 3-month T-bill rate, plus 1 to ensure a positive conditioning variable); Credit Spread (defined as difference between BAA and AAA-rated corporate bond yields); Stock Variance (defined as the sum of squared daily returns on the S&P 500); 1 + Net Equity Issuance (the ratio of 12-month moving sums of net issues by

NYSE-listed stocks to the total end-of-year market capitalizations of NYSE stocks, plus 1 to ensure a positive conditioning variable); 1 + Inflation (the Consumer Price Index, plus 1 to ensure a positive conditioning variable).

Denote by λ_0 the population mean of $\mathbb{E}_t[((R_{M,t,T_1} - R_{f,t,T_1}) - \tilde{r}_{M,t,T_1})z_t]$. To evaluate a bound's validity, we test the null hypothesis $\lambda_0 \geq 0$ against the alternative that λ_0 is unrestricted. To test tightness, we test the null hypothesis $\lambda_0 = 0$ against the alternative that $\lambda_0 \geq 0$.

Let $\bar{\lambda}$ denote the sample mean of $((R_{M,t,T_1} - R_{f,t,T_1}) - \tilde{r}_{M,t,T_1})z_t$, and Σ its sample covariance. Validity is tested using the statistic D_1 , defined as

$$D_1 = \min_{\lambda \geq 0} (\lambda - \bar{\lambda})\Sigma^{-1}(\lambda - \bar{\lambda}) \quad (\text{V19})$$

Under the null that $\lambda_0 \geq 0$, Back, Crotty, and Kazempour (2022) show that D_1 is asymptotically distributed as a mixture of chi-square distributions.

Under validity, tightness is tested using the statistic

$$D_2 = \hat{\lambda}'\Sigma^{-1}\hat{\lambda}, \quad (\text{V20})$$

where $\hat{\lambda}$ is the vector of λ which reaches the minimum in equation (V19). Under the null that $\lambda_0 = 0$, D_2 is also asymptotically distributed as a mixture of chi-square distributions.

Following Back, Crotty, and Kazempour (2022), we calculate finite sample p - values using Monte-Carlo simulations.

Table IAIII reports the results, for investment horizons T_N of one and two years. The statistic D_1 is zero for RP_{t,T_1}^{Log} and for RP_{t,T_1} for most forecast

horizons T_1 . It is positive for $RP_{t,T_1,1y}$ and $RP_{t,T_1,2y}$, but always below the 10% critical value. Validity is therefore never rejected. The statistic D_2 is the largest for RP_{t,T_1}^{Log} , smaller for RP_{t,T_1} , and even smaller for $RP_{t,T_1,1y}$ and $RP_{t,T_1,2y}$. Tightness is rejected for all bounds, except for $RP_{t,T_1,1y}$ and $RP_{t,T_1,2y}$ at $T_1 = 10$ days.

Table IAIII
Validity and tightness tests

We report test statistics for bound validity and bound tightness for different equity risk premium bounds. The test statistics D_1 and D_2 are defined in equation (V19) and (V20). The sample moments are means of weekly realized slackness and weekly realized slackness interacted with each of the variables from Welch and Goyal (2008) in z_t . The p -values are based on 100 Monte-Carlo simulations. Data are weekly from January 1996 to February 2023.

	RP_{t,T_1}^{Log}	RP_{t,T_1}	$RP_{t,T_1,1y}$	$RP_{t,T_1,2y}$
<i>Panel A: One week</i>				
D_1	0.00	0.00	0.41	1.44
p -value for validity	0.98	0.94	0.78	0.51
D_2	15.19	15.00	6.66	4.34
p -value for tightness	0.05	0.05	0.54	0.79
<i>Panel B: One month</i>				
D_1	0.00	0.66	2.09	4.24
p -value for validity	0.85	0.61	0.36	0.15
D_2	82.60	76.53	67.29	62.62
p -value for tightness	0.00	0.00	0.00	0.00
<i>Panel C: Two months</i>				
D_1	0.00	0.00	1.26	3.26
p -value for validity	0.86	0.87	0.46	0.18
D_2	225.41	221.85	195.53	175.32
p -value for tightness	0.00	0.00	0.00	0.00
<i>Panel D: One quarter</i>				
D_1	0.00	0.00	0.87	2.66
p -value for validity	0.83	0.75	0.45	0.24
D_2	405.06	395.76	356.34	320.61
p -value for tightness	0.00	0.00	0.00	0.00
<i>Panel E: Six months</i>				
D_1	0.00	0.00	1.00	2.48
p -value for validity	0.80	0.81	0.42	0.21
D_2	707.61	680.64	619.65	561.45
p -value for tightness	0.00	0.00	0.00	0.00

VI . Higher-order expansion

In this section, we investigate how higher-order moments contribute to the equity risk premium. We show that increasing the order of the approximation, thereby allowing for kurtosis preference, generates additional terms that contribute to the equity risk premium. For simplicity of notation, we assume that there is a single rebalancing date at T_1 , but all the derivations can be extended in a straightforward way to multiple rebalancing dates.

A. One-period SDF

Under no-arbitrage assumptions, a third-order Taylor expansion-series produces a one-period SDF in a three-date (two-period) economy of the form

$$\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \approx \frac{1 + z_{T_1} + z_{T_1}^v}{\mathbb{E}_t^* (1 + z_{T_1} + z_{T_1}^v)}, \quad (\text{VI1})$$

where

$$z_{T_1} = \frac{a_{1,t}}{R_{f,t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1}) + \frac{a_{2,t}}{R_{f,t,T_1}^2} (R_{M,t,T_1} - R_{f,t,T_1})^2 + \frac{a_{3,t}}{R_{f,t,T_1}^3} (R_{M,t,T_1} - R_{f,t,T_1})^3,$$

$$z_{T_1}^v = \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{M}_{T_1,T_N}^{*(2)} + \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{M}_{T_1,T_N}^{*(3)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} (R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{M}_{T_1,T_N}^{*(2)}. \quad (\text{VI2})$$

The coefficients are given by

$$a_{1,t} = \frac{1}{\tau_t}, \quad a_{2,t} = \frac{(1-\rho t)}{\tau_t^2}, \quad a_{3,t} = \frac{(\kappa t + 1 - 2\rho t)}{\tau_t^3}, \quad (\text{VI3})$$

and $a_{2,3,t} = 2a_{2,t} + 3a_{3,t}$. The risk aversion and skewness preference parameters are given by equations (7) and (8). The kurtosis tolerance parameters is given by

$$\kappa_t = \frac{1}{3!} \frac{u^{(4)} [W_t R_{f,t,T_N}] (u^{(1)} [W_t R_{f,t,T_N}])^2}{(u^{(2)} [W_t R_{f,t,T_N}])^3}. \quad (\text{VI4})$$

Proof. Consider the partial derivatives

$$\begin{aligned} f_{xxy} &= \frac{2}{(x_0)^2 y_0} \frac{(W_t x_0 y_0 u'')^2}{(u')^2} \left(2 - \frac{u''' u'}{(u'')^2} \right) \\ &\quad + \frac{1}{(x_0)^2 y_0} \left\{ 6 \frac{(W_t x_0 y_0)^3 u'' u' u'''}{(u')^3} - (W_t x_0 y_0)^3 \frac{u''''}{u'} - 6 \frac{(W_t x_0 y_0 u'')^3}{(u')^3} \right\}, \\ f_{xxx} &= \frac{y_0^3}{x_0^3} f_{yyy} = \frac{1}{(x_0)^3} \left(6 \frac{(W_t x_0 y_0)^3 u'' u'''}{(u')^2} - \frac{(W_t x_0 y_0)^3 u''''}{u'} - 6 \frac{(W_t x_0 y_0)^3 (u'')^3}{(u')^3} \right) \end{aligned} \quad (\text{VI5})$$

A third order Taylor expansion-series yields

$$\begin{aligned} f[x, y] &= f[x, y]^{2nd} \\ &\quad + \frac{1}{(x_0)^3} \frac{(\kappa_t + 1 - 2\rho_t)}{\tau_t^3} (x - x_0)^3 + \frac{1}{(y_0)^3} \frac{(\kappa_t + 1 - 2\rho_t)}{\tau_t^3} (y - y_0)^3 \\ &\quad + \frac{1}{(x_0)^2 y_0} \left(\frac{2(1 - \rho_t)}{\tau_t^2} + \frac{3(\kappa_t + 1 - 2\rho_t)}{\tau_t^3} \right) (x - x_0)^2 (y - y_0) \\ &\quad + \frac{1}{x_0 (y_0)^2} \left(\frac{2(1 - \rho_t)}{\tau_t^2} + \frac{3(\kappa_t + 1 - 2\rho_t)}{\tau_t^3} \right) (y - y_0)^2 (x - x_0) \end{aligned} \quad (\text{VI6})$$

where $f[x, y]^{2nd}$ is the second order Taylor expansion-series in Equation (I23).

Replacing x , x_0 , y , and y_0 by their expressions and using preference pa-

parameters a_1 , a_2 , and a_3 defined in Equation (VI3), we obtain,

$$\begin{aligned}
\mathbb{E}_{T_1}^* (f [x, y]) &= 1 + \frac{a_{1,t}}{R_{f,t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1}) + \frac{a_{1,t}}{R_{f,T_1,T_N}} (R_{f,T_1,T_N} - R_{f,t,T_1}) \\
&+ \frac{a_{2,t}}{(R_{f,t,T_1})^2} (R_{M,t,T_1} - R_{f,t,T_1})^2 + \frac{a_{2,t}}{(R_{f,T_1,T_N})^2} \mathbb{E}_{T_1}^* ((R_{M,T_1,T_N} - R_{f,T_1,T_N})^2) \\
&+ \frac{a_{1,t} + 2a_{2,t}}{R_{f,t,T_2}} (R_{M,t,T_1} - R_{f,t,T_1}) (R_{f,T_1,T_N} - R_{f,t,T_1}) \\
&+ \frac{a_{3,t}}{(R_{f,t,T_1})^3} (R_{M,t,T_1} - R_{f,t,T_1})^3 + \frac{a_{3,t}}{(R_{f,T_1,T_N})^3} \mathbb{E}_{T_1}^* ((R_{M,T_1,T_N} - R_{f,T_1,T_N})^3) \\
&+ \frac{2a_{2,t} + 3a_{3,t}}{(R_{f,t,T_1})^2 R_{f,T_1,T_N}} (R_{M,t,T_1} - R_{f,t,T_1})^2 (R_{f,T_1,T_N} - R_{f,t,T_1}) \\
&+ \frac{2a_{2,t} + 3a_{3,t}}{R_{f,t,T_1} (R_{f,T_1,T_N})^2} \mathbb{E}_{T_1}^* ((R_{M,T_1,T_N} - R_{f,T_1,T_N})^2) (R_{M,t,T_1} - R_{f,t,T_1}) \quad (\text{VI7})
\end{aligned}$$

which gives the desired result when interest rates are deterministic. □

When (VI2) is removed from the SDF specification (VI1), which corresponds to a static SDF in a one-period economy, the equity risk premium reduces to the expected excess return of Chabi-Yo and Loudis (2020). We refer to their bound to as RP_{t,T_1}^{3rd} .

Using this third-order expansion, we next derive the equity risk premium.

B. Equity risk premium

With the third-order Taylor expansion-series approach, Equation (VI1) depends on new terms such as risk-neutral skewness and cross-term between risk-neutral volatility and market excess return, in addition to risk-neutral variance. These terms introduce additional high-order leverage effects in the expected excess return decomposition. To find a closed-form expression for

the equity risk premium in terms of risk-neutral moments and high-order leverages, we first define high-order leverage effects under the risk-neutral measure as:

$$\mathbb{LES}_{t,T_1,T_N}^* = \text{COV}_t^* \left(R_{M,t,T_1} - R_{f,t,T_1}, \mathbb{M}_{T_1,T_N}^{*(3)} \right), \quad (\text{VI8})$$

$$\mathbb{LEK}_{t,T_1,T_N}^* = \text{COV}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2, \mathbb{M}_{T_1,T_N}^{*(2)} \right). \quad (\text{VI9})$$

We then show how the equity risk premium depends on these terms in the following Proposition.

Proposition 1: *Up to the third-order Taylor expansion-series of the inverse marginal utility, the one-period expected excess market return obeys the following decomposition*

$$\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1}) = \frac{\frac{a_{1,t}}{R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(2)} + \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(3)} + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{M}_{t,T_1}^{*(4)} + IH_{t,T_1,T_N}^{3rd}}{1 + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,t,T_1}^k} \mathbb{M}_{t,T_1}^{*(k)} + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,T_1,T_N}^k} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(k)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{LEV}_{t,T_1,T_N}^*}, \quad (\text{VI10})$$

where the intertemporal hedging component IH_{t,T_1,T_N}^{3rd} is given by

$$IH_{t,T_1,T_N}^{3rd} = \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{LEV}_{t,T_1,T_N}^* + \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{LES}_t^* + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \left(\mathbb{LEK}_{t,T_1,T_N}^* + \mathbb{M}_{t,T_1}^{*(2)} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \right), \quad (\text{VI11})$$

$a_{2,3,t} = 2a_{2,t} + 3a_{3,t}$ and the risk-neutral quantities $\mathbb{LEV}_{t,T_1,T_N}^*$, $\mathbb{M}_{T_i,T_j}^{*(k)}$, \mathbb{LES}_t^* and $\mathbb{LEK}_{t,T_1,T_N}^*$ are defined in Equations (10), (11), (VI8), and (VI9), respectively.

Proof. Using the inverse SDF (VII), we obtain

$$\begin{aligned}\mathbb{E}_t(R_{t,T_1} - R_{f,t,T_1}) &= \text{COV}_t^* \left(\frac{1 + z_{T_1} + z_{T_1}^v}{1 + \mathbb{E}_t^* z_{T_1} + \mathbb{E}_t^* z_{T_1}^v}, (R_{M,t,T_1} - R_{f,t,T_1}) \right) \\ &= \frac{\text{COV}_t^*(z_{T_1}, r_{M,t,T_1}) + \text{COV}_t^*(z_{T_1}^v, r_{M,t,T_1})}{1 + \mathbb{E}_t^* z_{T_1} + \mathbb{E}_t^* z_{T_1}^v}.\end{aligned}$$

Setting $r_{M,t,T_1} = R_{M,t,T_1} - R_{f,t,T_1}$ and using (VI.A) and (VI2), it follows that

$$\begin{aligned}\mathbb{E}_t^* z_{T_1} &= \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{E}_t^* r_{M,t,T_1}^2 + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{E}_t^* r_{M,t,T_1}^3 \\ \mathbb{E}_t^* z_{T_1}^v &= \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} + \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{E}_t^* r_{M,t,T_1} \mathbb{M}_{T_1,T_N}^{*(2)}\end{aligned}$$

Furthermore,

$$\begin{aligned}\mathbb{E}_t^* z_{T_1} (R_{M,t,T_1} - R_{f,t,T_1}) &= \frac{a_{1,t}}{R_{f,t,T_1}} \mathbb{E}_t^* r_{M,t,T_1}^2 + \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{E}_t^* r_{M,t,T_1}^3 + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{E}_t^* r_{M,t,T_1}^4 \\ &= \frac{a_{1,t}}{R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(2)} + \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(3)} + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{M}_{t,T_1}^{*(4)} \quad (\text{VI13})\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}_t^* z_{T_1}^v (R_{M,t,T_1} - R_{f,t,T_1}) &= \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \text{COV}_t^* \left(r_{M,t,T_1}, \mathbb{M}_{T_1,T_N}^{*(2)} \right) + \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \text{COV}_t^* \left(r_{M,t,T_1}, \mathbb{M}_{T_1,T_N}^{*(3)} \right) \\ &\quad + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \left(\text{COV}_t^* \left(r_{M,t,T_1}^2, \mathbb{M}_{T_1,T_N}^{*(2)} \right) + \mathbb{M}_{t,T_1}^{*(2)} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \right) \quad (\text{VI14})\end{aligned}$$

This ends the proof. \square

Furthermore, under Assumptions 1 and 2, our measure of the equity risk premium remains a lower bound for expected future market returns. The proof is similar to the one in the case of the second order approximation, see

Appendix II.E.

C. Physical variance

PROPOSITION IA1: *Up to a third-order expansion-series, under no-arbitrage conditions, the conditional variance of returns under the physical measure is a function of risk neutral return moments:*

$$\begin{aligned}\text{Var}_t &\equiv \mathbb{E}_t (R_{M,t,T_1} - \mathbb{E}_t R_{M,t,T_1})^2 \\ &= \mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1})^2 - (\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1}))^2\end{aligned}\quad (\text{VI15})$$

where $\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1})$ is given by Equation (VI10) and

$$\begin{aligned}\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1})^2 &= \frac{\begin{aligned} &\mathbb{M}_{t,T_1}^{*(2)} + \frac{a_{1,t}}{R_{f,t,T_1}} \mathbb{M}_{t,T_1}^{*(3)} + \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(4)} + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{M}_{t,T_1}^{*(5)} \\ &+ \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} \right) \\ &+ \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(3)} \right) \\ &+ \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^3 \mathbb{M}_{T_1,T_N}^{*(2)} \right) \end{aligned}}{\begin{aligned} &1 + \frac{a_{2,t}}{R_{f,t,T_1}^2} \mathbb{M}_{t,T_1}^{*(2)} + \frac{a_{3,t}}{R_{f,t,T_1}^3} \mathbb{M}_{t,T_1}^{*(3)} + \frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \\ &+ \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1}) \mathbb{M}_{T_1,T_N}^{*(2)} \right) \end{aligned}}.\end{aligned}\quad (\text{VI16})$$

Proof. The result follows from applying the third-order approximation of Equation (VII) and substituting it into

$$\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1})^2 = \mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} (R_{M,t,T_1} - R_{f,t,T_1})^2 \right).\quad (\text{VI17})$$

□

D. Conditional crash probability

We next express the conditional probability of a crash using a third-order Taylor expansion series for the inverse marginal utility. To derive this probability, we define additional truncated moments as

$$\mathbb{M}_{t,s}^*[\alpha] = \mathbb{E}_t^* \left(\mathbb{M}_{T_1, T_N}^{*(3)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right), \quad (\text{VI18})$$

$$\mathbb{M}_{t,sv}^*[\alpha] = \mathbb{E}_t^* \left(r_{M,t,T_1} \mathbb{M}_{T_1, T_N}^{*(2)} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right). \quad (\text{VI19})$$

Proposition 2: *Up to the third-order expansion-series of the inverse marginal utility, the conditional crash probability, $\Pi_{t,T_1, T_N}^{3rd}[\alpha] = P_t(R_{M,t,T_1} < \alpha)$, in a two-period (three-date) economy is*

$$\Pi_{t,T_1, T_N}^{3rd}[\alpha] = \frac{\left\{ \begin{aligned} & \mathbb{M}_{t,T_1}^{*(0)}[\alpha] + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,t,T_1}^k} \mathbb{M}_{t,T_1}^{*(k)}[\alpha] \\ & + \frac{a_{2,t}}{R_{f,T_1, T_N}^2} \mathbb{M}_{t,v}^*[\alpha] + \frac{a_{3,t}}{R_{f,T_1, T_N}^3} \mathbb{M}_{t,s}^*[\alpha] + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1, T_N}^2} \mathbb{M}_{t,sv}^*[\alpha] \end{aligned} \right\}}{1 + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,t,T_1}^k} \mathbb{M}_{t,T_1}^{*(k)} + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,T_1, T_N}^k} \mathbb{E}_t^* \mathbb{M}_{T_1, T_N}^{*(k)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1, T_N}^2} \mathbb{E}_t^* R_{M,t,T_1} \mathbb{M}_{T_1, T_N}^{*(2)}} \quad (\text{VI20})$$

where $a_{2,3,t} = 2a_{2,t} + 3a_{3,t}$.

Proof. The probability of crash is

$$\Pi_{t,T_1, T_N}^{3rd}[\alpha] = \mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t,T_1}}{m_{t,T_1}} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right)$$

We then replace the inverse SDF by its expression and obtain

$$\begin{aligned}
\Pi_{t,T_1,T_N}^{3rd}[\alpha] &= \frac{\mathbb{E}_t^* \left((1 + z_{T_1} + z_{T_1}^v) \mathbb{1}_{R_{M,t,T_1} < \alpha} \right)}{1 + \mathbb{E}_t^* z_{T_1} + \mathbb{E}_t^* z_{T_1}^v} \\
&= \frac{\mathbb{E}_t^* \left(\mathbb{1}_{R_{M,t,T_1} < \alpha} \right) + \mathbb{E}_t^* \left(z_{T_1} \mathbb{1}_{R_{M,t,T_1} < \alpha} \right) + \mathbb{E}_t^* \left(z_{T_1}^v \mathbb{1}_{R_{M,t,T_1} < \alpha} \right)}{1 + \mathbb{E}_t^* z_{T_1} + \mathbb{E}_t^* z_{T_1}^v} \\
&= \frac{\left\{ \begin{aligned} &\mathbb{M}_{t,T_1}^{*(0)}[\alpha] + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,t,T_1}^k} \mathbb{M}_{t,T_1}^{*(k)}[\alpha] + \\ &\frac{a_{2,t}}{R_{f,T_1,T_N}^2} \mathbb{M}_{t,v}^*[\alpha] + \frac{a_{3,t}}{R_{f,T_1,T_N}^3} \mathbb{M}_{t,s}^*[\alpha] + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{M}_{t,sv}^*[\alpha] \end{aligned} \right\}}{1 + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,t,T_1}^k} \mathbb{M}_{t,T_1}^{*(k)} + \sum_{k=2}^3 \frac{a_{k,t}}{R_{f,T_1,T_N}^k} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(k)} + \frac{a_{2,3,t}}{R_{f,t,T_1} R_{f,T_1,T_N}^2} \mathbb{E}_t^* R_{M,t,T_1} \mathbb{M}_{T_1,T_N}^{*(2)}}
\end{aligned}$$

This ends the proof. \square

E. Estimation of risk-neutral moments

Let us first derive a closed-form expression for \mathbb{LEK}_t^* .

$$\begin{aligned}
\mathbb{LEK}_{t,T_1,T_N}^* &= \text{COV}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2, \mathbb{M}_{T_1,T_N}^{*(2)} \right) \\
&= \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} \right) - \mathbb{M}_{t,T_1}^{*(2)} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} \quad (\text{VI21})
\end{aligned}$$

Using the projection in equation (32), we obtain

$$\mathbb{LEK}_{t,T_1,T_N}^* = \theta_{T_1,T_N} \left(\mathbb{M}_{t,T_1}^{*(4)} - \left(\mathbb{M}_{t,T_1}^{*(2)} \right)^2 \right) \quad (\text{VI22})$$

Let us now derive closed-form expressions for $\mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)}$ and \mathbb{LES}_t^* .

We can write $\mathbb{E}_t^* \mathbb{M}_{T_1, T_N}^{*(3)}$ and LES_t^* respectively as,

$$\begin{aligned} \mathbb{E}_t^* \mathbb{M}_{T_1, T_N}^{*(3)} &= \mathbb{E}_t^* \left((R_{M, T_1, T_N} - R_{f, T_1, T_N})^3 \right) \\ &= \mathbb{E}_t^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right) - 3R_{f, T_1, T_N} \mathbb{E}_t^* \mathbb{M}_{T_1, T_N}^{*(2)}, \quad (\text{VI23}) \end{aligned}$$

and

$$\begin{aligned} \text{LES}_t^* &= \text{COV}_t^* \left(R_{M, t, T_1}, \mathbb{M}_{T_1, T_N}^{*(3)} \right) \\ &= \text{COV}_t^* \left(R_{M, t, T_1}, \mathbb{E}_{T_1}^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right) - 3R_{f, T_1, T_N} \mathbb{M}_{T_1, T_N}^{*(2)} \right) \\ &= \text{COV}_t^* \left(R_{M, t, T_1}, \mathbb{E}_{T_1}^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right) \right) - 3R_{f, T_1, T_N} \text{COV}_t^* \left(R_{M, t, T_1}, \mathbb{M}_{T_1, T_N}^{*(2)} \right) \\ &= \text{COV}_t^* \left(R_{M, t, T_1}, \mathbb{E}_{T_1}^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right) \right) - 3R_{f, T_1, T_N} \text{LEV}_t^*. \quad (\text{VI24}) \end{aligned}$$

Let us assume that the term $\mathbb{E}_{T_1}^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right)$ is a nonlinear function g of $R_{M, t, T_1} - R_{f, t, T_1}$:

$$\mathbb{E}_{T_1}^* \left(R_{M, T_1, T_N}^3 - R_{f, T_1, T_N}^3 \right) = \gamma_{T_1, T_N} g[R_{M, t, T_1} - R_{f, t, T_1}] + v_{T_1}, \quad (\text{VI25})$$

with $\mathbb{E}_t^* (v_{T_1} | R_{M, t, T_1}) = \mathbb{E}_t^* (v_{T_1}) = 0$. Multiplying both sides of Equation (VI25) by R_{M, t, T_1}^3 and taking the time- t risk-neutral expectation, we obtain,

$$\gamma_{T_1, T_N} = \frac{\mathbb{M}_{t, T_N}^{*(3)} + 3R_{f, t, T_N} \mathbb{M}_{t, T_N}^{*(2)} - R_{f, T_1, T_N}^3 \left(\mathbb{M}_{t, T_1}^{*(3)} + 3R_{f, t, T_1} \mathbb{M}_{t, T_1}^{*(2)} \right)}{\mathbb{E}_t^* \left(R_{M, t, T_1}^3 g[R_{M, t, T_1} - R_{f, t, T_1}] \right)}. \quad (\text{VI26})$$

If we use $g[R_{M, t, T_1} - R_{f, t, T_1}] = R_{M, t, T_1}^3$, we obtain

$$\gamma_{T_1, T_N} = \frac{\mathbb{M}_{t, T_N}^{*(3)} + 3R_{f, t, T_N} \mathbb{M}_{t, T_N}^{*(2)} - R_{f, T_1, T_N}^3 \left(\mathbb{M}_{t, T_1}^{*(3)} + 3R_{f, t, T_1} \mathbb{M}_{t, T_1}^{*(2)} \right)}{\mathbb{E}_t^* \left(R_{M, t, T_1}^6 \right)}. \quad (\text{VI27})$$

Taking the expectation of (VI25) under the risk neutral measure,

$$\mathbb{E}_t^* (R_{M,T_1,T_N}^3) - R_{f,T_1,T_N}^3 = \gamma_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1}^3). \quad (\text{VI28})$$

Multiplying both sides of Equation (VI25) by R_{M,t,T_1} and taking the time- t risk-neutral expectation

$$\mathbb{E}_t^* (R_{M,t,T_1} R_{M,T_1,T_N}^3) = R_{f,t,T_1} R_{f,T_1,T_N}^3 + \gamma_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1}^4). \quad (\text{VI29})$$

Therefore, using Equations (VI23) and (VI24) we obtain $\mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)}$ and \mathbb{LES}_t^* as,

$$\mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} = \gamma_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1}^3) - 3R_{f,t,T_1} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)},$$

and

$$\mathbb{LES}_t^* = \gamma_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1}^4) - R_{f,t,T_1} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} - 3R_{f,t,T_1} R_{f,T_1,T_N} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} - 3R_{f,T_1,T_N} \mathbb{LEV}_t^*.$$

To compute the physical variance, we also need the following moments which we obtain using a similar approach:

$$\mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^3 (R_{M,T_1,T_N} - R_{f,T_1,T_N})^2 = \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^3 \mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N})^2. \quad (\text{VI30})$$

Using expression (32)

$$\mathbb{E}_{T_1}^* (R_{M,T_1,T_N} - R_{f,T_1,T_N})^2 = \theta_{T_1,T_N} (R_{M,t,T_1} - R_{f,t,T_1})^2 + \epsilon_{T_1}, \quad (\text{VI31})$$

it follows that

$$\mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^3 (R_{M,T_1,T_N} - R_{f,T_1,T_N})^2 = \theta_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^5. \quad (\text{VI32})$$

Finally,

$$\begin{aligned} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 (R_{M,T_1,T_N} - R_{f,T_1,T_N})^3 &= \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 R_{M,T_1,T_N}^3 \\ &\quad - R_{f,T_1,T_N}^3 \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 \\ &\quad + 3R_{f,T_1,T_N}^2 \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 R_{M,T_1,T_N} \\ &\quad - 3R_{f,T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 R_{M,T_1,T_N}^2. \end{aligned} \quad (\text{VI33})$$

This expression simplifies to

$$\begin{aligned} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 (R_{M,T_1,T_N} - R_{f,T_1,T_N})^3 &= \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{E}_{T_1}^* R_{M,T_1,T_N}^3 \\ &\quad + R_{f,T_1,T_N}^3 \mathbb{M}_{t,T_1}^{*(2)} \\ &\quad - 3R_{f,T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)}. \end{aligned} \quad (\text{VI34})$$

Since

$$\mathbb{E}_{T_1}^* R_{M,T_1,T_N}^3 = \gamma_{T_1,T_N} R_{M,t,T_1}^3, \quad (\text{VI35})$$

it follows that

$$\begin{aligned}
\mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 (R_{M,T_1,T_N} - R_{f,T_1,T_N})^3 &= \gamma_t \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 R_{M,t,T_1}^3 \right) \\
&\quad + R_{f,T_1,T_N}^3 \mathbb{M}_{t,T_1}^{*(2)} \\
&\quad - 3R_{f,T_1,T_N} \mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} \right)
\end{aligned} \tag{VI36}$$

where expression $\mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} \right)$ can be derived as follows:

$$(R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} = \theta_{T_1,T_N} (R_{M,t,T_1} - R_{f,t,T_1})^4 + (R_{M,t,T_1} - R_{f,t,T_1})^2 \varepsilon_{T_1} \tag{VI37}$$

and

$$\begin{aligned}
\mathbb{E}_t^* \left((R_{M,t,T_1} - R_{f,t,T_1})^2 \mathbb{M}_{T_1,T_N}^{*(2)} \right) &= \theta_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^4 + \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^2 \varepsilon_{T_1} \\
&= \theta_{T_1,T_N} \mathbb{E}_t^* (R_{M,t,T_1} - R_{f,t,T_1})^4
\end{aligned} \tag{VI38}$$

F. The case with consumption: Third-order Taylor expansion-series

Assume that (i) Assumptions 1 and 2 hold, (ii) the consumption-wealth ratio is positively related to the market return and (iii) the correlation between the square of the consumption wealth ratio and the market return is negative (condition reminiscent of market co-skewness). Under these assumptions, our measure of expected excess return is a lower bound to the true measure of market expected excess return.

A third-order Taylor expansion of $\frac{u'[\bar{W}_{t,T_N}]}{u'[W_{t,T_N}]}$ around $\mathbf{X} = \mathbf{X}_0$ gives

$$\begin{aligned}
\mathbf{G} = & 1 + (\mathbf{x} - \mathbf{x}_0) \frac{1}{\mathbf{x}_0} \frac{1}{\tau_t} + (\mathbf{y} - \mathbf{y}_0) \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} + (\mathbf{z} - \mathbf{z}_0) \frac{1}{\mathbf{z}_0} \frac{1}{\tau_t} + \frac{1}{\mathbf{x}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} (\mathbf{x} - \mathbf{x}_0)^2 \\
& + \frac{1}{\mathbf{y}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} (\mathbf{z} - \mathbf{z}_0)^2 + \frac{1}{\mathbf{x}_0 \mathbf{y}_0} \left(\frac{2(1 - \rho_t)}{\tau_t^2} \right) (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) \\
& + \left(\frac{\partial^2 \mathbf{G}}{\partial \mathbf{x} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{x} - \mathbf{x}_0) (\mathbf{z} - \mathbf{z}_0) + \left(\frac{\partial^2 \mathbf{G}}{\partial \mathbf{y} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{z} - \mathbf{z}_0) (\mathbf{y} - \mathbf{y}_0) \\
& + \frac{1}{\mathbf{x}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} (\mathbf{x} - \mathbf{x}_0)^3 + \frac{1}{\mathbf{z}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} (\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} (\mathbf{y} - \mathbf{y}_0)^3 \\
& + \frac{3}{3!} \frac{1}{\mathbf{x}_0^2 \mathbf{y}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0) \\
& + \frac{3}{3!} \frac{1}{\mathbf{y}_0^2 \mathbf{x}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0) \\
& + \frac{3}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{x}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{z} - \mathbf{z}_0)^2 (\mathbf{x} - \mathbf{x}_0) \\
& + \frac{3}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{z} - \mathbf{z}_0)^2 (\mathbf{y} - \mathbf{y}_0) \\
& + 6 \frac{1}{3!} \left(\frac{\partial^3 \mathbf{G}}{\partial \mathbf{x} \partial \mathbf{y} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{z} - \mathbf{z}_0) (\mathbf{y} - \mathbf{y}_0) (\mathbf{x} - \mathbf{x}_0) \\
& + 3 \frac{1}{3!} \left(\frac{\partial^3 \mathbf{G}}{\partial^2 \mathbf{x} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{z} - \mathbf{z}_0) + 3 \frac{1}{3!} \left(\frac{\partial^3 \mathbf{G}}{\partial^2 \mathbf{y} \partial \mathbf{z}} \right)_{\mathbf{x}=\mathbf{x}_0} (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{z} - \mathbf{z}_0) \quad (\text{VI39})
\end{aligned}$$

Therefore,

$$\begin{aligned}
v_{T_1} &= \mathbb{E}_{T_1}^* \mathbf{G} \\
&= 1 + (\mathbf{x} - \mathbf{x}_0) \frac{1}{\mathbf{x}_0} \frac{1}{\tau_t} + (\mathbf{y} - \mathbf{y}_0) \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} + \frac{1}{\mathbf{x}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} (\mathbf{x} - \mathbf{x}_0)^2 \\
&\quad + \frac{1}{\mathbf{y}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} \frac{(1 - \rho_t)}{\tau_t^2} \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\
&\quad + \frac{1}{\mathbf{x}_0 \mathbf{y}_0} \left(\frac{2(1 - \rho_t)}{\tau_t^2} \right) (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{1}{\mathbf{x}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} (\mathbf{x} - \mathbf{x}_0)^3 + \frac{1}{\mathbf{z}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3 \\
&\quad + \frac{1}{\mathbf{y}_0^3} \frac{(\kappa_t - 2\rho_t + 1)}{\tau_t^3} (\mathbf{y} - \mathbf{y}_0)^3 \\
&\quad + \frac{3}{3!} \frac{1}{\mathbf{x}_0^2 \mathbf{y}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{3}{3!} \frac{1}{\mathbf{y}_0^2 \mathbf{x}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0) \\
&\quad + \frac{3}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{x}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\
&\quad + \frac{3}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} \left(\frac{4(1 - \rho_t)}{\tau_t^2} + \frac{6(\kappa_t - 2\rho_t + 1)}{\tau_t^3} \right) (\mathbf{y} - \mathbf{y}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2
\end{aligned}$$

Using Equation (6) in the main text of the paper, it follows that

$$\begin{aligned}
v_{T_1} &= \mathbb{E}_{T_1}^* \mathbf{G} \\
&= 1 + (\mathbf{x} - \mathbf{x}_0) \frac{1}{\mathbf{x}_0} a_{1,t} + (\mathbf{y} - \mathbf{y}_0) \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} + \frac{1}{\mathbf{x}_0^2} a_{2,t} (\mathbf{x} - \mathbf{x}_0)^2 \\
&\quad + \frac{1}{\mathbf{y}_0^2} a_{2,t} (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 + \frac{2}{\mathbf{x}_0 \mathbf{y}_0} a_{2,t} (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{1}{\mathbf{x}_0^3} a_{3,t} (\mathbf{x} - \mathbf{x}_0)^3 + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} a_{3,t} (\mathbf{y} - \mathbf{y}_0)^3 \\
&\quad + \frac{6}{3!} \frac{1}{\mathbf{x}_0^2 \mathbf{y}_0} a_{2,3,t} (\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0) + \frac{6}{3!} \frac{1}{\mathbf{y}_0^2 \mathbf{x}_0} a_{2,3,t} (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0) \\
&\quad + \frac{6}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{x}_0} a_{2,3,t} (\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 + \frac{6}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} (\mathbf{y} - \mathbf{y}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2
\end{aligned}$$

We then compute the expected value $\mathbb{E}_t^* \nu_{T_1}$ to obtain

$$\begin{aligned}
\mathbb{E}_t^* \nu_{T_1} &= 1 + (\mathbf{x} - \mathbf{x}_0) \frac{1}{\mathbf{x}_0} a_{1,t} + \frac{1}{\mathbf{x}_0^2} a_{2,t} \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0)^2 \\
&+ \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 + \frac{2}{\mathbf{x}_0 \mathbf{y}_0} a_{2,t} \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) \\
&+ \frac{1}{\mathbf{x}_0^3} a_{3,t} \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0)^3 + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^3 \\
&+ \frac{1}{\mathbf{y}_0^2 \mathbf{x}_0} a_{2,3,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0) + \frac{1}{\mathbf{z}_0^2 \mathbf{x}_0} a_{2,3,t} \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\
&+ \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \text{COV}_t^* (\mathbf{y}, \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2). \tag{VI42}
\end{aligned}$$

Notice that $\mathbf{x} - \mathbf{x}_0 \leq 0$, and the following inequalities hold: $a_{1,t} > 0$, $a_{2,t} \leq 0$, $a_{3,t} \geq 0$, $a_{2,3,t} \geq 0$. Furthermore, $\mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0)^3 \leq 0$, $\mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^3 \leq 0$, $\mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0)^3 \leq 0$ and

$$\begin{aligned}
\mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0) &= \text{COV}_t^* (\mathbf{x} - \mathbf{x}_0, \mathbf{y}) \geq 0 \\
\mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0) &\leq 0 \text{ (because } \mathbf{x} - \mathbf{x}_0 \leq 0) \\
\mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 &\leq 0 \text{ (because } \mathbf{x} - \mathbf{x}_0 \leq 0) \\
\text{COV}_t^* (\mathbf{y}, \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2) &= \text{LEV}_t^* \leq 0. \tag{VI43}
\end{aligned}$$

This allows us to bound $\mathbb{E}_t^* \nu_{T_1}$ as

$$\begin{aligned}
\mathbb{E}_t^* \nu_{T_1} &\leq 1 + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\
&+ \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^3 \\
&+ \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \text{COV}_t^* (\mathbf{y}, \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2). \tag{VI44}
\end{aligned}$$

As a result,

$$\frac{1}{\mathbb{E}_t^* v_{T_1}} \geq \frac{1}{\left\{ \begin{aligned} &1 + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2 \\ &+ \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{E}_t^* (\mathbf{y} - \mathbf{y}_0)^3 \\ &+ \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \mathbb{C}\text{OV}_t^* (\mathbf{y}, \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2) \end{aligned} \right\}}. \quad (\text{VI45})$$

Next, our goal is to bound $\mathbb{C}\text{OV}_t^* (v_{T_1}, \mathbf{y} - \mathbf{y}_0) = \mathbb{C}\text{OV}_t^* (v_{T_1}, R_{M,t,T_1} - R_{f,t,T_1})$.

We use (VI41) to compute this covariance as

$$\begin{aligned} &\mathbb{C}\text{OV}_t^* (v_{T_1}, \mathbf{y} - \mathbf{y}_0) \\ = &\frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} \mathbb{V}\text{AR}_t^* (\mathbf{y}) + \frac{1}{\mathbf{x}_0^2} a_{2,t} \mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0)^2, \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{C}\text{OV}_t^* ((\mathbf{y} - \mathbf{y}_0)^2, \mathbf{y} - \mathbf{y}_0) + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{C}\text{OV}_t^* (\mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{2}{\mathbf{x}_0 \mathbf{y}_0} a_{2,t} \mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0), \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{1}{\mathbf{x}_0^3} a_{3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0)^3, \mathbf{y} - \mathbf{y}_0) + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{C}\text{OV}_t^* (\mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^3, \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{y} - \mathbf{y}_0)^3, \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{6}{3!} \frac{1}{\mathbf{x}_0^2 \mathbf{y}_0} a_{2,3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0), \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{6}{3!} \frac{1}{\mathbf{y}_0^2 \mathbf{x}_0} a_{2,3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0), \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{6}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{x}_0} a_{2,3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \\ &+ \frac{6}{3!} \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \mathbb{C}\text{OV}_t^* ((\mathbf{y} - \mathbf{y}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0). \end{aligned}$$

Notice that

$$\mathbb{C}\text{OV}_t^* ((\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0), \mathbf{y} - \mathbf{y}_0) = \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0) (\mathbf{y} - \mathbf{y}_0)^2 \leq 0 \quad (\text{since } \mathbf{x} \leq \mathbf{x}_0),$$

and

$$\mathbb{COV}_t^* ((\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0), \mathbf{y} - \mathbf{y}_0) = \mathbb{E}_t^* (\mathbf{x} - \mathbf{x}_0)^2 (\mathbf{y} - \mathbf{y}_0)^2 \geq 0.$$

We assume

$$\mathbb{COV}_t^* ((\mathbf{x} - \mathbf{x}_0)^2, \mathbf{y} - \mathbf{y}_0) \leq 0 \quad (\text{VI46})$$

and

$$\mathbb{COV}_t^* ((\mathbf{x} - \mathbf{x}_0)^3, \mathbf{y} - \mathbf{y}_0) \geq 0, \quad (\text{VI47})$$

$$\mathbb{COV}_t^* ((\mathbf{y} - \mathbf{y}_0)^2 (\mathbf{x} - \mathbf{x}_0), \mathbf{y} - \mathbf{y}_0) \geq 0, \quad (\text{VI48})$$

$$\mathbb{COV}_t^* ((\mathbf{x} - \mathbf{x}_0) \mathbb{E}_{T_1}^* (\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \geq 0. \quad (\text{VI49})$$

These conditions are reminiscent of the sign of coskewness and cokurtosis when random variables of interest are return. While $\mathbf{y} - \mathbf{y}_0$ and $\mathbf{z} - \mathbf{z}_0$ are realized excess returns, $\mathbf{x} - \mathbf{x}_0$ is a function of wealth-consumption ratio (see (II90)-(II92)). Because coskewness is negative (see Harvey and Siddique (2000)) and cokurtosis is positive (Dittmar (2002)) and the wealth-consumption ratio is positively correlated to the market return, one should expect (VI47)-(VI49) to hold.

Under conditions (VI46)-(VI49), it follows that

$$\begin{aligned}
\text{COV}_t^*(v_{T_1}, \mathbf{y} - \mathbf{y}_0) &\geq \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} \text{VAR}_t^*(\mathbf{y}) + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^*(\mathbf{y} - \mathbf{y}_0)^3 \\
&\quad + \frac{1}{\mathbf{z}_0^2} a_{2,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{1}{\mathbf{z}_0^3} a_{3,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^3, \mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{1}{\mathbf{y}_0^3} a_{3,t} \text{COV}_t^*((\mathbf{y} - \mathbf{y}_0)^3, \mathbf{y} - \mathbf{y}_0) \\
&\quad + \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \text{COV}_t^*((\mathbf{y} - \mathbf{y}_0) \mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0).
\end{aligned} \tag{VI50}$$

Combining (VI45) and (VI50) leads to

$$\begin{aligned}
\mathbb{E}_t(R_{M,t,T_1} - R_{f,t,T_1}) &= \frac{\text{COV}_t^*[v_{T_1}, R_{M,t,T_1}]}{\mathbb{E}_t^* v_{T_1}} \\
&\quad \left\{ \begin{array}{l} \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} \text{VAR}_t^*(\mathbf{y}) + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^*(\mathbf{y} - \mathbf{y}_0)^3 \\ \frac{1}{\mathbf{z}_0^2} a_{2,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \\ + \frac{1}{\mathbf{z}_0^3} a_{3,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^3, \mathbf{y} - \mathbf{y}_0) \\ + \frac{1}{\mathbf{y}_0^3} a_{3,t} \text{COV}_t^*((\mathbf{y} - \mathbf{y}_0)^3, \mathbf{y} - \mathbf{y}_0) \end{array} \right\} \\
&\geq \frac{\left\{ \begin{array}{l} 1 + \frac{1}{\mathbf{y}_0} a_{2,t} \mathbb{E}_t^*(\mathbf{y} - \mathbf{y}_0)^2 + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2 \\ + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_t^* \mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^3 + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{E}_t^*(\mathbf{y} - \mathbf{y}_0)^3 \\ + \frac{1}{\mathbf{z}_0^2 \mathbf{y}_0} a_{2,3,t} \text{COV}_t^*(\mathbf{y}, \mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2) \end{array} \right\}}{\left\{ \begin{array}{l} \frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} \text{VAR}_t^*(\mathbf{y}) + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{E}_t^*(\mathbf{y} - \mathbf{y}_0)^3 \\ \frac{1}{\mathbf{z}_0^2} a_{2,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^2, \mathbf{y} - \mathbf{y}_0) \\ + \frac{1}{\mathbf{z}_0^3} a_{3,t} \text{COV}_t^*(\mathbb{E}_{T_1}^*(\mathbf{z} - \mathbf{z}_0)^3, \mathbf{y} - \mathbf{y}_0) \\ + \frac{1}{\mathbf{y}_0^3} a_{3,t} \text{COV}_t^*((\mathbf{y} - \mathbf{y}_0)^3, \mathbf{y} - \mathbf{y}_0) \end{array} \right\}}
\end{aligned}$$

which simplifies to

$$\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1}) \geq \frac{\left(\frac{1}{\mathbf{y}_0} \frac{1}{\tau_t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{M}_{t,T_1}^{*(3)} + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{M}_{t,T_1}^{*(4)} \right) + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{L}\mathbb{E}\mathbb{V}_t^* + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{L}\mathbb{E}\mathbb{S}_t^*}{\left(1 + \frac{1}{\mathbf{y}_0^2} a_{2,t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{1}{\mathbf{z}_0^2} a_{2,t} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} + \frac{1}{\mathbf{y}_0^3} a_{3,t} \mathbb{M}_{t,T_1}^{*(3)} \right) + \frac{1}{\mathbf{z}_0^3} a_{3,t} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} + \frac{1}{\mathbf{z}_0^3 \mathbf{y}_0} a_{2,3,t} \mathbb{L}\mathbb{E}\mathbb{V}_t^*}$$

(VI51)

We, thereafter, replace \mathbf{y}_0 and \mathbf{z}_0 by their expressions

$$\mathbb{E}_t (R_{M,t,T_1} - R_{f,t,T_1}) \geq \frac{\left(\frac{1}{R_{f,t,T_1}} \frac{1}{\tau_t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{1}{R_{f,t,T_1}^2} a_{2,t} \mathbb{M}_{t,T_1}^{*(3)} + \frac{1}{R_{f,t,T_1}^3} a_{3,t} \mathbb{M}_{t,T_1}^{*(4)} \right) + \frac{1}{R_{f,T_1,T_N}^2} a_{2,t} \mathbb{L}\mathbb{E}\mathbb{V}_t^* + \frac{1}{R_{f,T_1,T_N}^3} a_{3,t} \mathbb{L}\mathbb{E}\mathbb{S}_t^*}{\left(1 + \frac{1}{R_{f,t,T_1}^2} a_{2,t} \mathbb{M}_{t,T_1}^{*(2)} + \frac{1}{R_{f,T_1,T_N}^2} a_{2,t} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(2)} + \frac{1}{R_{f,t,T_1}^3} a_{3,t} \mathbb{M}_{t,T_1}^{*(3)} \right) + \frac{1}{R_{f,T_1,T_N}^3} a_{3,t} \mathbb{E}_t^* \mathbb{M}_{T_1,T_N}^{*(3)} + \frac{1}{R_{f,T_1,T_N}^2 R_{f,t,T_1}} a_{2,3,t} \mathbb{L}\mathbb{E}\mathbb{V}_t^*}$$

(VI52)

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